Can Monetary Policy be Superior for Financial Stabilization?[1]

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Abstract

This paper assesses monetary policy in form of asset purchases as a financial stabilization instrument. We develop a monetary model with collateral constraints, where constrained efficiency is achieved by macroprudential regulation. Ex-post asset purchases in secondary markets and ex-post loan subsidies can outperform the latter. As the main novel contribution, we show that conducting welfare-enhancing asset purchases does not rely on tax revenues, in contrast to otherwise equivalent non-monetary policies. Asset purchases can be implemented independently of money supply and inflation targets, and can enhance efficiency irrespective of distributive effects or of inferior value extraction from central bank asset holdings.

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1 Introduction

Over the past two decades, central banks have implemented large-scale asset purchase programs during periods when interest rates approached the zero lower bound. Empirical studies provide clear evidence that central bank asset purchases altered various yields and asset prices, in particular, during and in the aftermath of the peak of the great financial crisis.[3] Asset purchase programs were further (re-)introduced or scaled-up at the onset of the Covid-19 pandemic, effectively mitigating price dislocations in financial markets during that period.[4] These observations suggest that ex-post monetary policy interventions are useful for correcting price movements that pose risks to financial stability. However, safeguarding financial stability is typically not the primary objective of central banks. This raises the question as to why other policy domains have not intervened ex-post to correct prices in financial markets. This paper addresses this question and presents a simple argument in favor of utilizing monetary policy for financial stabilization: central bank asset purchases can be implemented independently of tax revenues.

The analysis is conducted in a model with an essential role of money and with financial frictions, providing a rationale for macroprudential regulation in form of an ex-ante Pigouvian tax on debt (see Bianchi and Mendoza, 2018). As a reference point, we assume the availability of non-distortionary taxes. We establish that ex-post central bank asset purchases in secondary markets can implement constrained efficiency, just like macroprudential regulation. Asset purchases can however outperform the latter and can even implement the first best allocation. Yet, corrective price effects of asset purchases can equivalently be induced by a Pigouvian subsidy on credit supply that is compensated and financed by non-distortionary taxes. For the primary case of interest, we assume that taxes are unavailable, such that first best cannot be achieved and Pigouvian policies are not at the policy maker's disposal. As the main novel contribution of the paper, we show that central bank asset purchases can enhance efficiency relative to a competitive equilibrium without asset purchases irrespective of the availability of taxes.[5] We further show that the conclusions retain their validity when accounting for distributive effects or for an inferior ability of central banks to extract value from asset holdings – arguments often raised as constraints on the effectiveness of monetary policy interventions. The analysis builds on the

[3] See Dell'Ariccia et al. (2018) and Kuttner (2018), who survey evidence on effects of asset purchase programmes.

[4] See, for example, Haddad et al. (2021) for evidence based on US corporate bonds market data.

[5] To the best of our knowledge, implementation of unconventional monetary policy without taxes has not been explored in related studies, despite their beneficial effects being well-documented (see Section 2). In contrast, non-existence of taxes has been examined in studies on conventional monetary policy (see, e.g., Diaz-Gimenez et al., 2008, or Buera and Nicolini, 2020).

existence of sufficiently many central bank instruments. They not only facilitate the isolation of price effects of asset purchases and ensure independence from fiscal policy but also enable the attainment of non-financial monetary policy objectives. A real-world example for simultaneously promoting financial stability and pursuing primary objectives through distinct instruments has been the creation of the US Federal Reserve's Bank Term Funding Program in March 2023, coinciding with an upward trajectory in the Federal Funds rate target.

We develop a finite-horizon monetary model that serves two purposes.[6] Firstly, it explicitly describes the implementation of monetary policy to enable a clear differentiation between monetary and non-monetary policies. Secondly, it can be transformed into a tractable version, facilitating straightforward comparisons with studies on macroprudential regulation. In fact, central arguments of the paper would be rendered less comprehensible if the analysis were based on the latter version right from the beginning. Specifically, we assume that banks intermediate funds between households and that central bank money serves as the unique means to settle deposit transactions. Acknowledging central bank practice, reserves are exclusively supplied in exchange for eligible assets. These are treasury securities in regular open market operations where the central bank controls the terms of trade, i.e. the repo rate; the latter serving as the policy rate.[7] We assume that borrowers cannot commit to repay debt, such that bank loans will be collateralized by borrowers' assets. In addition to treasury open market operations, the central bank may further conduct asset purchases, i.e. purchases of bank loans against reserves. The analysis addresses three sources of inefficiency: Given that collateral constraints might be binding, intertemporal decisions of borrowers are distorted, leading to precautionary savings. Since private agents do not internalize the effect of their behavior on the collateral price, the price of pledgeable assets tends to be inefficiently low under binding collateral constraints. Moreover, costly central bank money supply, i.e. a positive policy rate, induces inefficiently low holdings of central bank money and deposits.

Before we derive the main novel result on asset purchases without taxes, we establish welfare-enhancing effects of macroprudential regulation and of asset purchases for the reference case where the availability of non-distortionary taxes aligns with the assumptions made in related studies (see Section 2). Taxes are used to repay initial public sector liabilities and to neutralize budgetary effects of corrective policies. Given that public sector solvency can then be ensured

[6] Similar to Dreze and Polemarchakis (2000), we apply reversed open market operations to overcome Hahn's (1965) paradox of maintaining a positive value for money in finite horizon models.

[7] In Appendix F, we demonstrate that introducing interest on reserves does not alter the results of the paper.

irrespective of central bank earnings, we assume that the central bank costlessly supplies reserves, like under the Friedman-rule (see, e.g., Chari and Kehoe, 1999). The allocation of commodities is then identical to an allocation under laissez faire in a corresponding non-monetary economy, closely relating to Davila and Korinek (2018) or Schabert (2024). For this case, we show that macroprudential regulation can address the pecuniary externality with regard to the collateral price and can implement a constrained efficient allocation (defined as in Stiglitz, 1982, or Davila et al., 2012), like in Jeanne and Korinek (2010), Bianchi (2011), or Bianchi and Mendoza (2018).

Next, we introduce ex-post central bank asset purchases in secondary markets as the primary policy instrument of focus. Specifically, the central bank offers buying collateralized loans from banks in states where collateral constraints bind. Given that reserves are not scarce, banks voluntarily sell loans to the central bank only at an above-market price.[8] This tends to increase the profitability of supplying (and selling) loans, driving down the loan rate demanded by competitive banks. Lower interest rate costs raise agents' willingness to borrow and thereby their willingness to pay for collateral. This tends to increase the price of pledgeable assets due to a higher collateral premium, i.e. the asset price element that measures the valuation of assets to serve as collateral, which is summarized by Bianchi and Mendoza (2018) as the "collateral effect" on asset pricing.[9] Via this mechanism, the central bank can raise the collateral price and can address the pecuniary externality, without inducing moral hazard.[10] Asset purchases can thereby implement constrained efficient allocations that pareto-dominate the constrained efficient allocation under ex-ante Pigouvian debt taxes. Moreover, asset purchases can elevate collateral prices to such an extent that the borrowing limit is never reached, leading to a competitive equilibrium that is characterized by the first best allocation. Yet, these price effects of asset purchases can equivalently be induced by an ex-post Pigouvian subsidy on loan supply that is funded and compensated by non-distortionary taxes, an optimal state-contingent credit market policy that corresponds to the ex-post debt subsidy in Katagiri et al. (2017) and Schabert (2024). Thus,

[8] The latter relates to the US Federal Reserves' Bank Term Funding Program, where purchased debt securities are valued at par value (instead of fair market value).

[9] This asset price component is also known as the "collateral value" (see Fostel and Geanakoplos, 2008) or the "collateralizability premium" (see Ai et al., 2020).

[10] The asset price effect of loan purchases differs from the mechanism that is responsible for ex-ante debt taxes to increase the collateral price as well as from effects of ex-post transfers of funds to borrowers or bailouts (as, for example, in Bianchi, 2016, or Jeanne and Korinek, 2020). In contrast to the latter, asset purchases do not affect borrowing ex-ante and do not create moral hazard, given that they are directed at lenders rather than borrowers (see also Bodenstein and Lorenzoni, 2018, or Jeanne and Korinek, 2020). In fact, corrective asset purchases induce a higher valuation of borrowers' assets in adverse states, whereas moral hazard can be created if bailouts (or debt reliefs) induce borrowers to value their wealth less (see Stavrakeva, 2020).

monetary policy interventions are so far not superior to non-monetary policy interventions.

The distinctive role of monetary policy becomes apparent when we posit the absence of non-distortionary taxes, a premise that can be justified empirically and theoretically (see e.g. Atkinson and Stiglitz, 1976, or Hammond, 1979). In this case, first best cannot be achieved and a Pigouvian subsidy that is equivalent to asset purchases cannot be implemented. To accentuate the argument, we assume that also other forms of taxation are unavailable, such that corrective policies cannot be financed with fiscal revenues. Given that public sector solvency requires revenues to be raised in an alternative way, a positive monetary policy rate is required, resulting in central bank interest earnings and scarcity of reserves. Banks are then principally willing to acquire reserves via asset purchases even when the purchase price of loans is lower than the market price. As the main novel result, we show that welfare-enhancing asset purchases can be implemented autonomously, without relying on tax revenues. From this perspective, asset purchases are a superior instrument for financial stabilization compared to equivalent non-monetary policies. While we focus on the ability of asset purchases to enhance efficiency by raising the collateral price, asset purchases can principally also affect social welfare via their impact on monetary aggregates. To isolate the corrective price effects of asset purchases, effects on monetary aggregates are neutralized via treasury open market operations.[11] We then show that efficiency can be enhanced relative to any competitive equilibrium without asset purchases. We further show that conducting asset purchases for financial stabilization does not necessitate the central bank to compromise other targets or objectives. In fact, it has sufficiently many instruments at its disposal to implement an inflation target independently of asset purchase programs.[12] The conclusions regarding the desirability of asset purchases would therefore remain valid even when nominal rigidities were introduced.[13]

Lastly, we examine whether asset purchases are associated with effects that limit their effectiveness, as posited in related studies (see, e.g., Jeanne and Korinek, 2020). In particular, we consider that the central bank is characterized by an inferior ability (compared to banks) to extract value from assets held under asset purchase programs. We refer to the fundamental imperfection that gives rise to the collateral constraint (i.e. limited commitment) and assume

[11] This relates for example to the sterilization of the US Federal Reserves' "Maturity Extension Program" introduced in 2011/2012, which left the overall size of the central bank balance sheet unchanged.

[12] For the analysis of inflation, we apply fiscal policy regimes (i.e. Ricardian and non-Ricardian) that either guarantee zero public sector liabilities in the terminal period or not (see e.g. Nakajima and Polemarchakis, 2005).

[13] Neutralization of inflation effects of unconventional monetary policy has also been applied in Schabert (2015) for the analysis of optimal monetary policy under sticky prices, in an economy devoid of financial markets frictions.

that the central bank can only seize a smaller fraction of borrowers' collateral than banks. We show that this property can be addressed with haircuts, which do not hinder the central bank to exert welfare-enhancing price effects via asset purchases. Moreover, we assess if distributive effects, which were ruled out by construction in the previous analysis, render asset purchases less desirable. The analysis of an augmented model shows that inefficiencies due to distributive effects can in fact be suitably addressed by raising the collateral price via asset purchases. Thus, neither distributive effects nor an inferior value extraction from central bank asset holdings invalidate the conclusions drawn above.

The remainder of the paper is organized as follows. Section 2 describes the related literature. In Section 3, we present the model. Section 4 presents results on welfare-enhancing ex-ante and ex-post policy interventions. Section 5 examines money and inflation under asset purchases. In Section 6, we augment the model to assess the robustness of the results. Section 7 concludes.

2 Related literature

Our paper is related to the two strands in the literature on the macroeconomic effects of unconventional monetary policies (i.e. public sector acquisitions of non-short-term treasury debt securities or private sector assets) and on macroprudential regulation under pecuniary externalities. None of these studies considers corrective price effects of asset purchases or examines the case where taxes are not available, which is the main novel contribution of this paper.

Regarding the first strand, several studies examine effects of unconventional monetary policies under financial market imperfections and price rigidities. Curdia and Woodford (2011) and Gertler and Karadi (2011) show that direct central bank lending to ultimate borrowers can be beneficial when financial intermediation via banks is costly. While ad-hoc costs of credit origination by the central bank renders financial intermediation exclusively by the central bank undesirable, costly central bank credit provision can be beneficial to mitigate financial crises. Chen et al. (2012) consider segmented financial markets and find that changing the composition of treasury debt as under US Federal Reserve large scale asset purchase programs during the financial crisis has moderate output and inflation effects. Del Negro et al. (2017) examine government purchases of equity in response to an adverse shock to assets' resaleability and show that introducing this policy has prevented a repeat of the Great Depression. Correia et al. (2021) compare welfare effects of tax-financed credit subsidies to those of monetary policy instruments in a cash-in-advance model with banking and costly enforcement. They show that credit subsidies are superior to a central bank interest rate policy, which is constrained by the zero lower bound, as well as to costly credit provision by the public sector modelled as in Gertler and Karadi (2011). Araújo et al. (2015) examine asset purchases in an economy with flexible prices and collateral constraints. They show that welfare effects are ambiguous, since purchases of assets that serve as collateral can loosen or tighten borrowing constraints. In contrast to our paper, they assume that money and bonds are perfect substitutes and that all central bank trades occur at market prices. Amador and Bianchi (2024) show that credit easing – modelled as purchases of capital by the government – can be benefical when banks face a run, whereas it can push more banks to default under fundamentally driven crises. None of these papers considers pecuniary externalities due to financial frictions or provides a comparison with macroprudential policies.

The second strand of the literature, to which our paper is related, focusses on a financial amplification mechanism via effects of pecuniary externalities induced by collateral constraints (see Lorenzoni, 2008, Jeanne and Korinek, 2010, 2019, Bianchi 2011, Benigno et al., 2016 & 2023, Schmitt-Grohe and Uribe, 2017, Bianchi and Mendoza, 2018, Davila and Korinek, 2018, or Korinek, 2018).[14] A common conclusion drawn in these studies is that the amplification of adverse shocks and welfare losses under potentially binding collateral constraints can be mitigated by macroprudential policies that reduce borrowing ex-ante, which has established the view that a laissez-faire equilibrium in these types of economies is characterized by overborrowing.[15] Like in these studies, we show that macroprudential regulation can enhance social welfare and can implement a constraint efficient allocation. Within the same class of models, Benigno et al. (2016) show the desirability of ex-post taxes on non-tradables that raise the collateral price, and Bianchi (2016) shows that debt reliefs lead to welfare gains, which both relate to our analysis of ex-post policies. Using a variant of Jeanne and Korinek's (2010) model where agents internalize collateral services of pledgeable assets, Katagiri et al. (2017) show that an ex-post debt subsidy can implement first best by raising the collateral price via the collateral premium such that the borrowing limit is not binding. Schabert (2024) reaffirms this result in two models, with one of them being Davila and Korinek's (2018) model, and further shows that non-state-contingent saving subsidies can outperform ex-ante debt taxes by mitigating distributive effects of pecuniary externalities. None of these studies considers monetary policy.

[14] Davila and Korinek (2018) further examine distributive effects – which we adress in Section 6 – in a comprehensive analysis of effects induced by pecuniary externalities based on financial frictions.

[15] The generality of the latter conclusion has been questioned by Benigno et al. (2013) and Schmitt-Grohe and Uribe (2021).

Few studies merge the two aforementioned topics and are most closely related to our paper. Bornstein and Lorenzoni (2018) and Jeanne and Korinek (2020) analyze the relation between ex-ante and ex-post policies. The latter are identified with monetary policy interventions. While Jeanne and Korinek's (2020) specification of ex-ante policies in form of a Pigouvian tax on debt corresponds to ours, the crucial difference to our analysis is the specification of ex-post policy. For the latter, they distinguish untargeted from targeted ex-post "liquidity provisions", i.e. open market purchases of borrowers' assets or loans proportional to the recipient's debt; both being financed by real funds borrowed from lenders/depositors.[16] Given that ex-post liquidity provisions are not associated with financial constraints, they can principally implement first best. For the analysis of the optimal mix of ex-ante and ex-post policies, Jeanne and Korinek (2020) consider ad-hoc social costs of liquidity provisions. They show that costly ex-post policies do not obviate ex-ante regulation, while a more generous liquidity provision allows relaxing ex-ante regulation. This main conclusion relates to Bornstein and Lorenzoni (2018), who develop a framework with pre-set nominal prices and aggregate demand externalities. They show that macroprudential regulation is unnecessary when the central bank sets the real interest rate in a state contingent way, whereas macroprudential regulation is useful when monetary policy is not fully state contingent. They further examine asset purchases, which are associated with ad-hoc inefficiency costs, and conclude that they "seem to be substitutes for ex-ante macroprudential policy" (p. 274). Like in Jeanne and Korinek (2020), purchases of assets are financed by funds raised from lenders via taxes and repaid to lenders. In both studies, policy therefore enforces that funds are transferred from lender/depositors to borrowers, such that borrowing against collateral is reduced. In contrast to our model, neither Bornstein and Lorenzoni (2018) nor Jeanne and Korinek (2020) consider any role for money.

An essential role for money is assumed in Woodford (2016) and Chi et al. (2024), which both disregard central bank acquisition of non-treasury debt or borrowers' assets. Woodford (2016) considers interest rate policy, reserve requirements, and "quantitative easing", which is identified with an extension of the central bank balance sheet via purchases of long-term treasuries. The focus of the analysis is whether these independent dimensions of monetary policies increase or decrease financial stability risk. To address this question, Woodford (2016) embeds Stein's (2012) fire sale model in a dynamic general equilibrium model where financial crises

[16] They list several alternative interpretations of ex-post policy liquidity provisions, namely, open market purchases, uniform lump-sum transfers, or interest rate cuts for untargeted interventions, and discount window lending, debt relief, or recapitalizations for targeted interventions.

are exogenously triggered. The paper shows that lowering interest rates tend to raise financial stability risks whereas "quantitative easing policies should not increase risks to financial stability, but rather should tend to reduce them" (p. 153). Chi et al. (2024) augment the model of Bianchi (2011) by introducing banks which require reserves. In contrast to Woodford (2016) and related to our approach, Chi et al. (2024) consider that households instead of banks are financially constrained. By increasing interest on reserves when borrowing constraints bind monetary policy incentivizes banks to expand holdings of reserves, which are financed by increased foreign debt. With this increase in public sector revenues the government can reduce income taxes imposed on constrained households, which – combined with a capital control policy – implements a constraint efficient allocation. Contrary to the policies in Woodford (2016) and Chi et al. (2024), the main monetary policy measure considered in our paper, i.e. price-correcting asset purchases, neither increases the central bank balance sheet nor alters interest rate on reserves.

3 The model

In this section, we present a finite-horizon model developed to serve two particular purposes. On one hand, the model enables comparisons with research on macroprudential regulation. On the other hand, we acknowledge that central bank money is essential for the settlement of transactions and we explicitly specify monetary policy implementation, enabling a clear differentiation between monetary and non-monetary policies. Precisely, model central bank money supply against eligible assets in secondary markets.[17] We thereby deviate from a textbook-style money supply specification, which is not suited for the purpose of this paper since it counterfactually predicts asset purchases to have no price effects (see Appendix E). To embed central bank operations in a realistic structure, we include banks in the model, though they are not relevant for allocation of commodities. The economy further consists of households, who borrow from and deposit funds at banks, and a government. Due to limited commitment, households can borrow only against collateral. Deposits and bank loans are traded in nominal terms with money serving as the unit of account. Banks further hold government bonds, which are eligible for regular open market operations. In addition, the central bank can supply money via asset purchases, i.e., purchases of collateralized loans in secondary markets.

The analysis starts with the reference case where the availability of non-distortionary taxes

[17] The specification of central bank operations closely relates to Schabert (2015), who analyses optimal monetary policy under frictionless financial markets under sticky prices.

aligns with the assumptions made in related studies (see Section 2). Precisely, we consider lump-sum taxes/transfers. Given that income is exogenously determined in our model, an income tax would also be non-distortionary. As we assume exogeneity of income solely for simplicity, we disregard income taxes, which typically induce distortions when income is endogenous. We subsequently examine the primary case of interest where taxes are unavailable. For the efficiency analysis in Section 4, we will restrict attention to a three-period version of the model with quasi-linear utility, facilitating comparisons with related studies (see Bornstein and Lorenzoni, 2018, and Jeanne and Korinek, 2020). Before, the model is specified in a more general way in order to establish that main equilibrium properties, i.e. (ir-)relevance of monetary policy and equilibrium separability, also hold for a less restricted time horizon and for non-linear utility. The latter will be applied in Section 6 for the analysis of distributive effects.

3.1 Households

Agents/households i live from period $t = 0$ to $t = T$. There exist two types of agents $i \in (b, l)$. For both types, there is a continuum of agents of mass one. In each period, agents receive a stochastic income, i.e. a stochastic endowment of non-durable goods $y_{i,t}$. We restrict agents' initial wealth and endowments to ensure that one type of agents always acts as a borrower (b) and one as a lender (l). Utility of all agents increases with consumption $c_{i,t}$ of a non-durable good. We further assume that utility can increase with the housing stock $h_{i,t}$ and holdings of deposits $d_{i,t}$ (in real terms), as a short-cut for modelling transaction services of deposits. The instantaneous utility function $u_{i,t}$,

$$u_{i,t} = u(c_{i,t}, d_{i,t}, h_{i,t}), \quad \text{for } t \in [t, T-1] \text{ and } \; u_{i,T} = u(c_{i,T}, h_{i,T}), \tag{1}$$

is assumed to be separable and concave in all arguments. Following conventional textbook specifications of money-in-the-utility function (see e.g. Woodford, 2003), there exists a satiation level in real deposits at a finite positive value $\overline{d} > 0$, such that $u_d(\overline{d}) = 0$ and $u_d(d_{i,t}) > 0$ if $d_{i,t} < \overline{d}$. Corresponding to studies on fire sales, borrowers are assumed to have a superior use for pledgeable assets. Here, housing is serves as collateral and we assume that housing provides utility only to borrowers, i.e. $u_{l,t} = u(c_{l,t}, d_{l,t})$ for $t \in [t, T-1]$ and $u_{l,T} = u(c_{l,T})$, while they do not to hold other assets, i.e. $u_{b,t} = u(c_{b,t}, h_{b,t})$.

Income y_i realizes at the beginning of each period and depend on the state s. From $t = 0$ to $t = T - 1$, agents of type b draw relatively low realizations of income and borrow from banks, whereas agents of type l draw higher income realizations and deposit funds at banks. Income of agents b in $t = T$ is sufficiently high to repay debt. We assume that loans and deposits are contracted in nominal terms. The interest rate on bank loans $L_{b,t}$ is R_t^L and on deposits is R_t^D. The budget constraint of type-b-agents is given by

$$P_t y_{b,t} \geq -L_{b,t} + R_{t-1}^L L_{b,t-1} + P_t c_{b,t} + P_t q_t \left(h_{b,t} - h_{b,t-1}\right) + P_t \tau_{b,t}, \tag{2}$$

where $\tau_{i,t}$ denotes type-specific lump-sum transfers/taxes, P_t the price of non-durables, and q_t the real price of housing. The budget constraint of agents of type l, who will never borrow from banks and will not hold housing, is given by

$$P_t y_{l,t} \geq D_{l,t} - R_{t-1}^D D_{l,t-1} + P_t c_{l,t} - P_t \omega_{l,t} + P_t \tau_{l,t}, \tag{3}$$

where $D_{l,t} = P_t d_{l,t}$ denotes deposits and $\omega_{l,t}$ profits of banks, owned by type-l-agents.

A central element is a financial constraint, which can be microfounded by limited commitment and the possibility of debt renegotiation as follows: We assume that borrowers can threaten to repudiate the debt contract and that lenders protect themselves by collateralizing borrowers' housing. Following a repudiation, lenders can seize a fraction z of borrowers' housing and can sell it at the current price $P_t q_t$ in the housing market. We consider the case where borrowers have all the bargaining power and are able to negotiate the loan down to the liquidation value of their housing (see Hart and Moore, 1994). Lenders take this possibility into account, such that debt repayment does not exceed the value of the seizable collateral. Hence, debt $L_{b,t} > 0$ of a borrower b with housing h_b is constrained by

$$L_{b,t} \leq z P_t q_t h_{b,t}, \tag{4}$$

where the fraction $z \in (0,1)$ relies on enforcement and measures lenders' ability to extract value from assets (see Section 6). Maximizing lifetime utility $E \sum_{t=0}^{T} \beta^t u_{i,t}$, where E denotes the expectation operator, subject to the budget constraint (2) and the collateral constraint (4), leads to the following optimality conditions for borrowers for $t = 0$ to $t = T-1$

$$u'\left(c_{b,t}\right) q_t = u'\left(h_{b,t}\right) + \beta E_t u'\left(c_{b,t+1}\right) q_{t+1} + \{\zeta_{b,t} z q_t\}, \tag{5}$$

$$u'\left(c_{b,t}\right) = \beta R_t^L E_t u'\left(c_{b,t+1}\right) \pi_{t+1}^{-1} + \zeta_{b,t}, \tag{6}$$

as well as $u'\left(c_{b,T}\right) q_T = u'\left(h_{b,T}\right)$ and $l_{b,T} = L_{b,T}/P_T = 0$, where $\pi_{t+1} = P_{t+1}/P_t$ denotes the inflation rate and $\zeta_{b,t} \geq 0$ denotes the multiplier on the collateral constraint (4). The term in the curly brackets in (5) summarizes the collateral premium of the asset, i.e. the valuation of the asset to serve as collateral (see, e.g., Bianchi and Mendoza, 2018).[18] The first order conditions of lenders, who are restricted by the budget constraint (3), are $\lambda_{l,t} = u'(c_{l,t})$,

$$u'(c_{l,t}) = u'(d_{l,t}) + \beta R_t^D E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} \tag{7}$$

for $t = 0$ to $t = T - 1$, and $d_{l,T} = 0$, where $\lambda_{l,t}$ denotes the multiplier on (3).

3.2 Banks

There is a continuum of perfectly competitive banks $j \in [0, 1]$, which are equally endowed and will behave in an identical way. They receive deposits $D_{j,t}$ from type-l-agents and supply loans $L_{j,t}$ to type-b-agents. They hold short-term government bonds (i.e., treasury bills) $B_{j,t}$, which are issued at the period-t-price $1/R_t$. Banks further hold central bank money in form of reserves $M_{j,t}$ because of their unique ability to settle deposit transactions. Reserves are supplied via open market transactions, which are carried out as outright transactions or as temporary sales or purchases (repos). Both types of open market transactions are introduced to account for real-world practice. For both, treasury bills serve as eligible assets and the price of reserves in treasury open market operations (the repo rate) is R_t^m, which serves as the main policy rate. Specifically, reserves supplied to bank j in treasury open market operations $I_{j,t}^B$ satisfy

$$I_{j,t}^B \leq \widetilde{\kappa}_t^B B_{j,t-1}/R_t^m. \tag{8}$$

For simplicity, we abstract from modelling auctions and introduce $\widetilde{\kappa}_t^B \in [0, 1]$ as a share of randomly selected treasuries that are accepted as eligible assets. Alternatively, $\widetilde{\kappa}_t^B$ can be interpreted as the allotment rate, where $\widetilde{\kappa}_t^B = 1$ implies full allotment. In addition to these regular open market operations, we consider the possibility that the central bank temporarily purchases loans from banks, which we summarize as <u>asset purchases</u>. Specifically, the central bank announces to offer reserves under repos in exchange for a share $\widetilde{\kappa}_t^L \in [0, 1]$ of bank loans that are randomly selected to avoid differential treatment. The purchase price offered by the central bank equals $1/R_t^A$:

$$I_{j,t}^L \leq \widetilde{\kappa}_t^L L_{j,t}/R_t^A. \tag{9}$$

Notably, the purchase price $1/R_t^A$ might differ from the central bank price of bonds $1/R_t^m$ and from the market price of loans (= 1), and can be higher or lower than the latter. We assume

[18] Notably, the collateral premium would be absent if the borrowing limit were solely a function of aggregate variables, as for example specified in Jeanne and Korinek (2020). We will examine consequences of the latter type of borrowing constraints for policy effects in the subsequent analysis.

that central bank money is required for the settlement of deposit transactions, which are not explicitly modelled. For this, banks hold central bank money equal to a fraction $\widetilde{\mu} \in (0,1)$ of deposits (see Appendix A)

$$\widetilde{\mu} D_{j,t} \leq I_{j,t}^{B} + I_{j,t}^{L} + M_{j,t-1}. \tag{10}$$

Given that bank j transferred T-bills to the central bank under outright sales and that it repurchases a fraction of T-bills, $B_{j,t}^{R} = R_t^m M_{j,t}^{R}$, from the central bank, bank j's money holdings equal $M_{j,t-1} - R_t^m M_{j,t}^R + I_{j,t}^B + I_{j,t}^L$. Banks can further trade reserves among each other at the end of each period, after repos are settled. Thus, bank j's profits $P_t \omega_{j,t}$ are given by

$$\begin{aligned} P_t \omega_{j,t} = & D_{j,t} - R_{t-1}^D D_{j,t-1} - L_{j,t} + R_{t-1}^L L_{j,t-1} - B_{j,t}/R_t + B_{j,t-1} - M_{j,t} + M_{j,t-1} \\ & - I_{j,t}^B (R_t^m - 1) - I_{j,t}^L (R_t^A - 1), \end{aligned} \tag{11}$$

where the second line of (11) gives the costs of reserves acquisition. Notably, the last term tends to raise profits if the purchase price of loans, $1/R_t^A$, exceeds the market price, 1. The aggregate stock of reserves only changes via money supply operations (8) and (9), while demand deposits can be created subject to (10). We abstract from interest payments on reserves (R_t^R) to avoid complicating the notation even further. In fact, the efficiency results/conditions that refer to the rates R_t^m and R_t^A would then apply to their differences to R_t^R (see Appendix F).

We assume that bankers maximize profits, $E_t \sum_{k=0}^{T} p_{t,t+k} \omega_{j,t+k}$, where $p_{t,t+k}$ denotes the shareholders' stochastic discount factor $p_{t,t+k} = \beta^k \lambda_{j,t+k}/\lambda_{j,t}$, subject to (8)-(11). The first order conditions with respect to loans, deposits, reserves from treasury open market operations and from asset purchases, holdings of treasury securities and of money for $t = 0$ to $t = T-1$ are

$$\lambda_{j,t} = \beta E_t R_t^L \lambda_{j,t+1} \pi_{t+1}^{-1} + \kappa_{j,t}^L \widetilde{\kappa}_t^L / R_t^A, \tag{12}$$

$$\lambda_{j,t} = \beta E_t \left(R_t^D \lambda_{j,t+1} \right) \pi_{t+1}^{-1} + \mu_{j,t} \widetilde{\mu}, \tag{13}$$

$$\mu_{j,t} = \kappa_{j,t}^B + \lambda_{j,t} (R_t^m - 1), \tag{14}$$

$$\kappa_{j,t}^L = \mu_{j,t} - \lambda_{j,t} (R_t^A - 1), \tag{15}$$

$$\lambda_{j,t}/R_t = \beta E_t \lambda_{j,t+1} \pi_{t+1}^{-1} + \beta E_t \kappa_{j,t+1}^B \widetilde{\kappa}_{t+1}^B \pi_{t+1}^{-1} / R_{t+1}^m, \tag{16}$$

$$\lambda_{j,t} = \beta E_t \pi_{t+1}^{-1} \left(\lambda_{j,t+1} + \mu_{j,t+1} \right), \tag{17}$$

where $\kappa_{j,t}^B \geq 0$, $\kappa_{j,t}^L \geq 0$ and $\mu_{j,t} \geq 0$ denote the multipliers on (8), (9), and (10), respectively. The first order condition (12) shows that asset purchases tend to reduce the loan rate if the money supply constraint (9) is binding, $\kappa_{j,t}^L > 0$, which either requires reserves to be scarce ($\mu_{j,t} > 0$) or an above-market-price for loan purchases, $1/R_t^A > 1$ (see 15). The conditions (14) and (16) show that the bond rate is analogously affected by a binding money supply constraint for treasury operations, $\kappa_{j,t}^B > 0$. Moreover, the first order conditions (14), (16), and (17) relate the treasury rate to the expected monetary policy rate. Under full allotment $\widetilde{\kappa}_t^B = 1$, they imply

$$\beta E_t \pi_{t+1}^{-1} \left(\lambda_{j,t+1} + \mu_{j,t+1} \right) = \beta E_t \pi_{t+1}^{-1} [(\mu_{j,t+1} + \lambda_{j,t+1}) \cdot R_t / R_{t+1}^m], \tag{18}$$

and thus $R_t = R_{t+1}^m$ for a non-state-contingent monetary policy rate. Given that banks are perfectly competitive, bank profits (11) will be equal to zero in equilibrium.

In the terminal period T, banks will neither supply loans $L_{j,T} = 0$ nor create new deposits $D_{j,T} = 0$. Likewise, their money and bond holdings will equal zero at the end of period $T : M_{j,T} = B_{j,T} = 0$. Accordingly, there will be no asset purchases in period T, $I_{T,t}^L = 0$, and treasury open market operations are conducted as outright sales of bonds held by the central bank against reserves, $I_{j,T}^B = -M_{j,T-1}$. This relates to Dreze and Polemarchakis's (2000) specification and overcomes Hahn's (1965) paradox of maintaining a positive value for money under a finite horizon.

3.3 Public sector

The treasury issues bills B_t^g, i.e. one-period bonds, at the price $1/R_t$ and receives remittances τ_t^m from the central bank. For the first part of the analysis, we assume that the treasury has type-specific lump-sum taxes/transfers $P_t \tau_{i,t}$ at its disposal. This assumption is dropped in the subsequent part of the analysis, where we assess policy options when lump-sum taxes/transfers are not available. The treasury's budget constraint reads

$$(B_t^g / R_t) + P_t \tau_{b,t} + P_t \tau_{l,t} + P_t \tau_t^m = B_{t-1}^g. \tag{19}$$

Notably, lump-sum taxes/transfers $\tau_{i,t}$ will serve two purposes: Firstly, they are a source of revenues to repay initial public sector liabilities. Secondly, they might finance and compensate policy interventions that will be introduced below. For the analysis of inflation in Section 5, we characterize tax regimes in a more specific way and refer to Ricardian and non-Ricardian regimes (see e.g. Benhabib et al., 2001).

The central bank supplies money in open market operations either outright or temporarily via repos against treasuries, M_t and M_t^R, where $I_t^B = M_t - M_{t-1} + M_t^R$. The central bank can further increase the supply of money by purchasing loans from banks, I_t^L. For simplicity, we only consider temporary purchases of loans, i.e. the central bank supplies money under repos against loans. At the beginning of each period, its holdings of treasuries and the stock of outstanding money are given by B_{t-1}^c and M_{t-1}. It then receives money in exchange for treasuries and, eventually, for loans. Before the asset market opens, where the central bank rolls over maturing assets, repos in terms of treasuries and loans are settled. Its budget constraint therefore reads $(B_t^c/R_t) - B_{t-1}^c + P_t\tau_t^m = M_t - M_{t-1} + (R_t^m - 1) I_t^B + \left(R_t^A - 1\right) I_t^L$. Remittances to the treasury $P_t\tau_t^m$ consist of interest earnings from money supply as well as from asset holdings,[19]

$$P_t\tau_t^m = (R_t^m - 1)(M_t - M_{t-1}) + (R_t^m - 1) M_t^R + \left(R_t^A - 1\right) I_t^L + (1 - 1/R_t) B_t^c. \tag{20}$$

Substituting out remittances in the central bank budget constraint shows that central bank asset holdings evolve according to $B_t^c - B_{t-1}^c = M_t - M_{t-1}$. Further assuming that initial values satisfy $B_{-1}^c = M_{-1}$, implies for the central bank balance sheet $B_t^c = M_t$. The central bank has five instruments at its disposal: As the main (conventional) policy instruments, it sets the policy rate R_t^m and can decide how much money to supply against treasuries, $\widetilde{\kappa}_t^B \in (0,1]$. In addition, it can offer money at the price $1/R_t^A$ in exchange for a fraction $\widetilde{\kappa}_t^L \in [0,1]$ of bank loans. With these four instruments the central bank can influence market prices, e.g. $1/R_t$ or $1/R_t^L$ (see 12 and 16), and monetary aggregates, e.g. M_t or D_t (see 8, 9 and 10). Notably, the availability of the two instruments $\widetilde{\kappa}_t^L$ and $\widetilde{\kappa}_t^B$ enables the central bank to neutralize the impact on monetary aggregates of one type of money supply (see Section 5). Finally, the central bank can choose how much money to supply outright or temporarily via repos in exchange for treasuries, by controlling the ratio of treasury repos to outright purchases $\Omega_t \geq 0 : M_t^R = \Omega_t M_t$. While the central bank can principally adjust its earnings via this instrument, its it not required for the derivation of the main results in the subsequent sections.

3.4 Equilibrium

Before we identify welfare-enhancing policies in an analytical way, for which we apply simplifying assumptions, we summarize some main properties of the model. Throughout, we assume that initial wealth and stochastic income are identical for all agents of one type (b or l), and ensure that agents never switch roles. Initial endowment and income of lenders suffice to meet borrowers' demand for external funds, while period T income of borrowers suffices to repay debt. In equilibrium, agents' optimal plans are satisfied and prices adjust such that all markets

[19] Notably, remittances τ_t^m can in principle also be negative in the reference case, where we assume that non-distortionary taxes are available and that $R_t^m = 1$. For the main case of interest, where taxes are not available and $R_t^m > 1$, non-negativity of τ_t^m can be assumed without invalidating the results on welfare-enhancing asset purchases.

clear: $\int l_{j,t}dj = \int l_{b,t}db$, $\int d_{j,t}dj = \int d_{l,t}dl$, $h = \int h_{b,t}db$, $\int y_{b,t}db + \int y_{l,t}dl = \int c_{b,t}db + \int c_{l,t}dl$, $m_t = \int m_{j,t}dj$, $m_t^R = \int (i_{j,t}^B - m_{j,t} + m_{j,t-1}\pi_t^{-1})dj$, $b_t = \int b_{j,t}dj$, and $b_t^g = b_t^c + b_t$, where $m_{i,t}^H = M_{i,t}^H/P_t$, $m_t^R = M_t^R/P_t$, $b_{i,t} = B_{i,t}/P_t$, $b_t = B_t/P_t$, $b_t^c = B_t^c/P_t$, and $b_t^T = B_t^T/P_t$. Given that lenders are shareholders of banks, $\lambda_{j,t} = \lambda_{l,t}$ holds. Consolidating the budget constraints of the central bank and of the treasury, gives

$$(b_t/R_t) + \tau_{b,t} + \tau_{l,t} + R_t^m\left(m_t - m_{t-1}\pi_t^{-1}\right) + \left(R_t^m - 1\right)\Omega_t m_t + \left(R_t^A - 1\right)i_t^L = b_{t-1}\pi_t^{-1}, \quad (21)$$

where we used $b_t^g = b_t^c + b_t$. Together with the central bank balance sheet $B_t^c = M_t$, the latter implies $b_t^g - b_t = m_t$. Iterating (21) forward and applying the terminal conditions $m_T = b_T = 0$, gives[20]

$$\begin{aligned}&\left(b_{-1} + R_0^m m_{-1}\right)/\pi_0 - \sum_{t=0}^{T}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)\left(\tau_{l,t} + \tau_{b,t}\right) \quad (22)\\ =&\sum_{t=0}^{T-1}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)\left\{\left[R_t^m - R_{t+1}^m/R_t\right]m_t + \left[\left(R_t^m - 1\right)\Omega_t m_t\right] + \left(R_t^A - 1\right)i_t^L\right\}.\end{aligned}$$

The intertemporal budget constraint (22) shows that initial liabilities have to be repaid by tax revenues and central bank interest earnings. The latter are summarized by the terms in the curly brackets on the RHS of (22). The first term in square brackets depends on the price of money in terms of bonds charged by the central bank. The costs of money supplied outright in t equals R_t^m, whereas money holdings reduce costs in $t+1$ by R_{t+1}^m. The value of the latter discounted with the bond rate, R_{t+1}^m/R_t, tends to be equal or smaller than one. Under full allotment $\widetilde{\kappa}_t^B = 1$ and a non-state-dependent policy rate, the bond price R_t equals tomorrows policy rate R_{t+1}^m (see 18). For smaller values of $\widetilde{\kappa}_t^B$, the bond rate tends to be larger (see 16). Thus, the first term in square brackets tends to be positive for $R_t^m > 1$. The second term in square brackets measures interest earnings from repos, which are strictly positive for $R_t^m > 1$. Thus, (22) can be satisfied for $R_t^m > 1$, even when taxes are not available, $\tau_{i,t} = 0$. The last term on the RHS of (22) measures earnings/costs of asset purchases.

Using (21), bank $j's$ profits (11) can be written as $P_t\omega_{j,t} = D_{j,t} - R_{t-1}^D D_{j,t-1} - L_{j,t} + R_{t-1}^L L_{j,t-1} + P_t\tau_{b,t} + P_t\tau_{l,t}$. Substituting out profits with the latter, the budget constraint of a

[20] A more conventional version of the intertemporal budget constraint of the public sector emerges for $R_t^m = R_t^A = 1$, which corresponds to textbook-style monetary models (see (93) in Appendix E).

representative depositor (3), who acts as the ultimate lender, implies

$$y_{l,t} = l_{b,t} - R^L_{t-1}\pi_t^{-1} l_{b,t-1} + c_{l,t} - \tau_{b,t}. \tag{23}$$

We can then define a competitive equilibrium for two types of agents as follows, where we neglect the reference to individual banks (j), for convenience.

Definition 1 *A competitive equilibrium of the economy with two types of agents consists of a set of sequences* $\{c_{b,t}, c_{l,t}, h_{b,t}, q_t, \zeta_{b,t}, \mu_t, \kappa_t^L, l_{b,t}, d_{l,t}, \pi_t, \kappa_t^B, R_t, R_t^L, R_t^D, m_t, b_t, i_t^B, i_t^L\}_{t=0}^{T-1}$ *and* $\{c_{b,T}, c_{l,T}, q_T, \pi_T\}$ *satisfying* $y_{l,t} + y_{b,t} = c_{b,t} + c_{l,t}$, *(21), and (23) for* $t \in \{0, T\}$, $h_{b,t} = h$, *(5)-(7),*

$$l_{b,t} \leq z q_t h_{b,t}, \text{ if } \zeta_{b,t} = 0, \text{ or } l_{b,t} = z q_t h_{b,t}, \text{ if } \zeta_{b,t} > 0, \tag{24}$$

$$u'(c_{l,t}) = \beta R_t^L E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} + \kappa_t^L \widetilde{\kappa}_t^L / R_t^A, \tag{25}$$

$$u'(c_{l,t}) = \beta R_t^D E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} + \widetilde{\mu}\mu_t, \tag{26}$$

$$\kappa_t^L = \mu_t - u'(c_{l,t})(R_t^A - 1) \geq 0, \tag{27}$$

$$\kappa_t^B = \mu_t - u'(c_{l,t})(R_t^m - 1) \geq 0, \tag{28}$$

$$u'(c_{l,t})/R_t = \beta E_t u'(c_{l,t+1}) \pi_{t+1}^{-1} + \beta E_t \kappa_{t+1}^B \widetilde{\kappa}_{t+1}^B \pi_{t+1}^{-1} / R_{t+1}^m, \tag{29}$$

$$u'(c_{l,t}) = \beta E_t \pi_{t+1}^{-1} (u'(c_{l,t+1}) + \mu_{t+1}), \tag{30}$$

$$i_t^B \leq \widetilde{\kappa}_t^B b_{t-1} \pi_t^{-1} / R_t^m \text{ if } \kappa_t^B = 0, \text{ or } i_t^B = \widetilde{\kappa}_t^B b_{t-1} \pi_t^{-1} / R_t^m \text{ if } \kappa_t^B > 0, \tag{31}$$

$$i_t^L \leq \widetilde{\kappa}_t^L l_{b,t} / R_t^A \text{ if } \kappa_t^L = 0, \text{ or } i_t^L = \widetilde{\kappa}_t^L l_{b,t} / R_t^A \text{ if } \kappa_t^L > 0, \tag{32}$$

$$\widetilde{\mu} d_t \leq i_t^B + i_t^L + m_{t-1}\pi_t^{-1} \text{ if } \mu_t = 0, \text{ or } \widetilde{\mu} d_t = i_t^B + i_t^L + m_{t-1}\pi_t^{-1} \text{ if } \mu_t > 0, \tag{33}$$

$$i_t^B = m_t - m_{t-1}\pi_t^{-1} + \Omega_t m_t, \tag{34}$$

for $t \in \{0, T-1\}$, *where* $\mu_T = \kappa_T^B = 0$, $u'(c_{b,T}) q_T = u'(h)$, *and (22), given* $\{y_{l,t}, y_{b,t}\}_{t=0}^T$, *monetary policy* $\{R_t^m\}_{t=0}^T$ *and* $\{\widetilde{\kappa}_t^B, \widetilde{\kappa}_t^L, R_t^A, \Omega_t\}_{t=0}^{T-1}$, *fiscal policy* $\{\tau_{l,t}, \tau_{b,t}\}_{t=0}^T$, *and initial values* $R_{-1}^L > 0$, $l_{b,-1} \geq 0$, $b_{-1} > 0$, *and* $m_{-1} > 0$.

As indicated by Definition 1, the competitive equilibrium including initial inflation π_0 can be fully determined under suited policies. According to (34), reversed open market operations are conducted in the terminal period T, where $m_T = 0$. Further details are given in Section 5.

Combining the lenders' optimality condition for deposits (7) with the one of banks (26), shows that the multiplier μ_t on the banks' liquidity constraint (10) satisfies

$$u'(d_{l,t}) = \mu_t \widetilde{\mu}. \tag{35}$$

Thus, lenders are satiated with deposits, $d_{l,t} = \overline{d}$, when the multiplier on the banks' liquidity constraint (10) equals zero, $\mu_t = 0$. If the central bank sets $R_t^m = 1$, the exchange of treasuries against money in open market operations is costless. Banks are then not unwilling to hold more central bank money than required by (10) and are thus indifferent with regard to the size of open market transactions. If money is further supplied under full allotment in treasury open market operations, $\widetilde{\kappa}_t^B = 1$, and no asset purchases are offered by the central bank, $\widetilde{\kappa}_t^L = 0$, the liquidity constraint (10) and the money supply constraint (8) are slack, $\mu_t = \kappa_t^B = 0$, deposit demand is satiated, and all interest rates equal zero.

Corollary 1 *If $R_t^m = 1$, $\widetilde{\kappa}_t^B = 1$, and $\widetilde{\kappa}_t^L = 0$, the liquidity constraint (10) and the money supply constraint (8) are slack, while deposit demand is satiated $d_{l,t} = \overline{d}$ and all interest rates are equal to zero $R_t = R_t^L = R_t^D = 1$.*

Proof. See Appendix. ■

Under the case considered in Corollary 1, monetary policy is irrelevant for the allocation of commodities, which is identical to the allocation of a corresponding non-monetary economy. For comparability with related studies, we refer to this case as a laissez faire equilibrium.

Definition 2 *A laissez faire equilibrium is a competitive equilibrium under $R_t^m = \widetilde{\kappa}_t^B = 1$ and $\widetilde{\kappa}_t^L = 0$.*

Now consider that the central bank offers purchases of loans, $\widetilde{\kappa}_t^L > 0$. Given that reserves are not scarce under $R^m = 1$, banks would be indifferent between selling loans or not if the central bank offers the market price of loans after issuance $1/R_t^A = 1$. If however the central bank offers loan purchases at an above-market-price $1/R_t^A > 1$, banks are willing to sell as much loans as possible and the multiplier κ_t^L on (9) is strictly positive $\kappa_t^L = u'(c_{l,t})(1 - R_t^A) > 0$ for $\mu_t = 0$ (see 27). Potential profits from asset sales are then driven down to zero by reductions of the loan rate R_t^L, which takes a value below one (see 25). Thus, asset purchases can be non-neutral even when central bank money is not scarce, $\mu_t = 0$.

A particularly useful case, on which we will focus below, is a utility function with constant marginal utility of deposits. In this case, we can exploit the possibility to separate two subsets of equilibrium objects. The competitive equilibrium can be redefined such that the allocation of commodities is independent of conventional monetary policy, specifically, of the policy rate R_t^m. In contrast, the asset purchases instruments R_t^A and $\widetilde{\kappa}_t^L$ can affect the allocation of commodities via their impact on the loan rate.

Corollary 2 *For $\partial u'(d_{l,t})/\partial d_{l,t} = 0$, a competitive equilibrium is a set of sequences $\{c_{b,t},\ c_{l,t},\ q_t,\ h_{b,t},\ \zeta_{b,t},\ \mu_t,\ \kappa_t^L,\ l_{b,t},\ r_{t+1}^L = R_t^L \pi_{t+1}^{-1},\ r_{t+1}^D = R_t^D \pi_{t+1}^{-1}\}_{t=0}^{T-1}$ and $\{c_{b,T}, c_{l,T}, q_T\}$ satisfying $y_{l,t} + y_{b,t} = c_{b,t} + c_{l,t}$ and (23) for $t \in \{0, T\}$, $h = h_{b,t}$, (5)-(7), (24)-(27) for $t \in \{0, T-1\}$, and $u'(c_{b,T})\, q_T = u'(h)$, given $\{R_t^A,\ \widetilde{\kappa}_t^L\}_{t=0}^{T-1}$, $\{\tau_{b,t}\}_{t=0}^{T}$, and initial values $\pi_0 > 0$, $R_{-1}^L > 0$ and $l_{b,-1} \geq 0$.*

Proof. See Appendix. ■

For a constant marginal utility of deposits, which includes the case of satiated deposit demand $u'(d_{l,t}) = 0$, the competitive equilibrium can thus be summarized in a way that is equivalent to a non-monetary version of the economy, except of the possibility of loan purchases by the central bank. Specifically, equilibrium objects summarized in Corollary 2 are independent of the policy rate R_t^m, the bond rate R_t, deposits d_t, bonds b_t, and money m_t, $i_{j,t}^B$, and $i_{j,t}^L$. Notably, determination of the real loan rate in period 0, $R_{-1}^L \pi_0^{-1}$, is based on an initial nominal loan rate R_{-1}^L and the initial inflation rate π_0, which is taken as given in Corollary 2. As summarized in the following corollary, initial inflation, the subsequent inflation rates and other equilibrium objects related to monetary policy, e.g. monetary aggregates and the bond rate, are functions of the remaining monetary instruments $\widetilde{\kappa}_t^B$, Ω_t, and R_t^m. Given that the latter instruments do not affect the allocation of consumption, housing, and loans (see Corollary 2), they can freely be set, for example, in accordance with (other) policy objectives. In Section 5, we show how inflation rates are determined.

Corollary 3 *Suppose that* $\partial u'(d_{l,t}) / \partial d_{l,t} = 0$. *For given equilibrium sequences* $\{l_{b,t}, \mu_t\}_{t=0}^{T-1}$, $\{c_{l,t}\}_{t=0}^{T}$ *, and* $\{\widetilde{\kappa}_t^L, R_t^A\}_{t=0}^{T-1}$ *and* $\{\tau_{b,t}\}_{t=0}^{T}$*, the set of sequences* $\{d_t, \pi_t, R_t, \kappa_t^B, m_t, b_t, i_t^B, i_t^L\}_{t=0}^{T-1}$ *and* π_T *can be determined by (21), (28)-(34) for* $t \in \{0, T-1\}$*, (22), and policies* $\{\widetilde{\kappa}_t^B, \Omega_t, R_t^m\}_{t=0}^{T-1}$ *and* R_T^m*, and* $\{\tau_{l,t}\}_{t=0}^{T}$.

Proof. See Appendix. ■

Notably, the separation of monetary aggregates will be particularly helpful for the analysis of the case where lump-sum taxes/transfers are not available and the policy rate exceeds zero, $R_t^m > 1$. In this case, deposit demand will not be satiated, such that social welfare depends on the prevailing level of deposits held by lenders. Below we will establish that asset purchases can alter the loan rate (see 12). Yet, they can also change the supply of money (see 9). We will show that the former price effect is crucial for the welfare-enhancing role of asset purchases, whereas the latter, i.e. the money supply effect of asset purchases, might affect deposits and can potentially matter for non-financial targets of the central bank. Corollary 3 implies that monetary aggregates, including deposits, are further affected by other monetary policy instruments. Below we will show that adjustments in treasury money supply operations can be used to neutralize the money supply effects of asset purchases with regard to the allocation of commodities (see Section 4.5), and to implement inflation targets independently of asset purchases (see Section 5).

4 Efficiency analysis

In this section, we present results on welfare-enhancing policies. To facilitate the derivation of analytical results and comparisons with related studies, we restrict our attention to three periods and impose some common simplifying assumptions on preferences and endowments (see e.g. Davila and Korinek, 2018), which will hold throughout the analysis unless stated otherwise. We start the efficiency analysis by summarizing some main properties of the first best allocation and the laissez faire equilibrium. We continue by considering a well-established macroprudential policy, namely an ex-ante Pigouvian tax on debt, and show that it can address the pecuniary externality and can implement a constrained efficient allocation. As the main policy instrument of interest, we then examine effects of central bank loan purchases. For the idealized case where lump-sum taxes/transfers are available, we show that asset purchases are equivalent to a Pigouvian loan subsidy. They can implement a constrained efficient allocation that pareto-dominates the allocation under ex-ante Pigouvian debt taxes and even the first best allocation. We then turn to the primary case of interest where taxes and therefore equivalent Pigouvian policies are not available. As the main novel result of the paper, we then show that introducing asset purchases can enhance efficiency compared to any competitive equilibrium without asset purchases irrespective of the availability of taxes.

4.1 Simplifying assumptions

For the remainder of the analysis, we focus on the common specification with $T = 2$, implying three periods, $t = 0$, 1, and 2. The two types of agents are endowed with different initial debt/wealth levels. There is no uncertainty in the periods 0 and 2. Initial endowment with wealth/debt and income (in terms of non-durables) is assumed to ensure that agents b borrow $0 < l_{b,0} < zq_0h_{b,0}$ in an unconstrained way in period 0 and that debt can be repaid. In period 1, income is random. We consider two equally likely states $s \in (c, u)$. Income can either take the values $y_{b,1}(u)$ and $y_{l,1}(u)$ such that borrowing in period 1 is *u*nconstrained, or the values $y_{b,1}(c)$ and $y_{l,1}(c)$ such that $y_{b,1}(c) < y_{l,1}(c)$ and borrowing is *c*onstrained. Aggregate income satisfies $y_{b,0} + y_{l,0} = y_{b,2} + y_{l,2} = y$, where $y = 2$ and $y_{b,0} \leq y_{l,0}$, and might be risky in $t = 1$, $y_{b,1}(u) + y_{l,1}(u) = y_1(u) \geq y_{b,1}(c) + y_{l,1}(c) = y_1(c)$ where $y_1(u) + y_1(c) = y$. We introduce type- and time-specific utility functions, and assume that borrowers do not face net taxes/transfers.

Assumption 1 *Agents live for three periods and have preferences satisfying*

$$u_{b,t} = \log(c_{b,t}) + \log(h_{b,t}), \text{ for } t \in (0,1), \text{ and } u_{b,2} = c_{b,2} + \log(h_{b,2}),$$
$$u_{l,t} = c_{l,t} + f(d_{l,t}), \text{ for } t \in (0,1), \text{ and } u_{l,2} = c_{l,2},$$

with $f_d(\overline{d}) = 0$ and $f_d(d_{l,t}) > 0$ if $d_{l,t} < \overline{d}$. *There are no net taxes/transfers on/to borrowers.*

Assumption 2 *Initial net wealth of borrowers* $n_{b,0} = y_{b,0} - R^L_{-1}\pi_0^{-1}l_{b,-1}$ *ensures that* $l_{b,0} \geq 0$ *and that the borrowing constraint is slack in period 1,* $\zeta_{b,0} = 0$*. Borrowers' income in state c in period 1 is small enough that the borrowing constraint is binding under laissez faire.*

The assumptions of a three-period time horizon and quasi-linear utility follow common practice in related studies on macroprudential regulation (see Lorenzoni, 2008, Bornstein and Lorenzoni, 2018, Davila and Korinek, 2018, or Jeanne and Korinek, 2010, 2020). Linearity induces irrelevance of the allocation of non-durable goods in the final period, such that a redistributive motive of a social planer cannot be rationalized. Linear utility of lenders further implies that the real loan rate r^L_t in a laissez faire equilibrium satisfies

$$E_0 r^L_1 = r^L_2 = 1/\beta, \quad \text{with } r^L_t = R^L_{t-1}\pi_t^{-1}, \tag{36}$$

while it can be lowered via asset purchases (see 25). Under linear utility in period 2, there are no distributive effects with regard to debt/savings. For the analysis of distributive effects under asset purchases in Section 6.2, we therefore apply non-linear utility functions. It should further be noted that we will consider lump-sum transfers/taxes on/to borrowers only as compensations for distortionary taxes that are introduced to correct prices under Pigouvian policies, such that borrowers do not face net taxes or transfers (see Assumption 1).

4.2 First best and laissez faire

In this section, we describe the first best allocation and the laissez faire allocation under Assumptions 1 and 2. Given that borrowers' and lenders' utility are linear in the terminal period (see Assumption 1), any allocation of available resources between both types of agents in period 2 is first best. This implies that there is no justification for assigning different welfare weights to borrowers and lenders. Without loss of generality, agents' welfare weights can therefore be set equal to one,[21] such that social welfare is given by

[21] A social welfare function $W = E\sum_{t=0}^{2}\beta^t(\phi_b u_{b,t} + \phi_l u_{l,t})$ with welfare weights ϕ_i for $i \in \{b, l\}$ would imply the optimal allocation to satisfy $E\frac{u'(c_{l,t})}{u'(c_{b,t})} = \frac{\phi_b}{\phi_l}$ $\forall t \in [0,2]$ and thus $\phi_b = \phi_l$, since $u'(c_{i,2}) = 1$.

$$W = E\sum_{t=0}^{2}\beta^t\left(u_{b,t}+u_{l,t}\right). \quad (37)$$

A social planer who aims at maximizing (37) subject to the resource constraint, $y_{l,t}+y_{b,t} = c_{b,t}+c_{l,t}$, will choose consumptions levels of agents such that the marginal utilities of agents are identical and deposits are held at the satiation level. Like in Jeanne and Korinek (2020), the first best allocation is identical to the allocation of an unconstrained laissez faire equilibrium, i.e. a laissez faire equilibrium where the collateral constraint never binds. This allocation and the associated asset price q_1 will serve as a reference case in the subsequent analysis.

Proposition 1 *Under Assumption 1, the first best allocation satisfies $d_{l,0} = d_{l,1} = \overline{d}$, and*

$$c_{i,0}^{fb} = c_{i,1}^{fb}(s) = 1, \quad (38)$$

for $i \in \{b,l\}$ and $s \in \{u,c\}$. This allocation is implemented in an unconstrained laissez faire equilibrium, where the associated asset price q_1^{fb} satisfies $q_1^{fb} = h^{-1}(1+\beta)$ in all states s.

Proof. See Appendix. ■

While a laissez faire equilibrium without borrowing constraints leads to the first best allocation, a laissez faire allocation under constrained borrowing (see Assumption 2) is inefficient. Specifically, borrowers' consumption in $t=0$ is lower than under first best due a potentially binding borrowing constraint, leading to precautionary saving. When the borrowing constraint binds in $t=1$, consumption in $t=1$ is also lower than under first best as well as the housing price q_1.

Proposition 2 *Under Assumptions 1 and 2, a laissez faire allocation satisfies*

$$c_{b,0}^{lf} < 1, \quad c_{b,1}^{lf}(u) = 1, \; c_{b,1}^{lf}(c) < 1,$$

and the asset price

$$q_1^{lf}(c) = \frac{(1+\beta)\,h^{-1}}{(1-z)(1/c_{b,1}(c))+z}, \quad (39)$$

where $q_1^{lf}(u) = q_1^{fb}$ and $q_1^{lf}(c) = q_1(c_{b,1}^{lf}(c)) < q_1^{fb}$.

Proof. See Appendix. ■

The collateral (housing) price q_1 relates to borrowers' consumption $c_{b,1}$ under a binding borrowing constraint (see 39), which is not internalized by individual agents. Efficiency can then be enhanced under higher consumption $c_{b,1}$ and a higher collateral price level; the latter enabling agents to increase borrowing. This effect of the pecuniary externality can be addressed by a social planer with corrective policies, which will be subsequently examined. Throughout the remainder of the analysis, Assumptions 1 and 2 hold unless stated otherwise.

4.3 Macroprudential regulation and constrained efficiency

As a well-established policy intervention under collateral constraints (see Jeanne and Korinek, 2010, or Bianchi, 2011), we examine a Pigouvian tax on debt $\tau^d_{b,t}$ that is imposed before the borrowing constraint might be binding, a policy that is summarized as macroprudential regulation (see Bianchi and Mendoza, 2018).[22] To avoid non-corrective effects of this tax, type-specific tax revenues are rebated in a type-specific and lump-sum way $\tau_{b,0} = -\tau^d_{b,0} l_{b,0}$. Then, the set of equilibrium conditions is exclusively affected by the corrective debt tax via the borrowing condition

$$(1 - \tau^d_{b,0}) c^{-1}_{b,0} = \beta E_0 (r^L_1 c^{-1}_{b,1}), \tag{40}$$

which replaces the corresponding optimality condition of borrowers under laissez faire, i.e. $c^{-1}_{b,0} = \beta E_0 (r^L_1 c^{-1}_{b,1})$. Notably, the equilibrium relation between the asset price q_1 and borrowers' consumption $c_{b,1}$ under laissez faire (39) is unaffected by the debt tax. It can be shown that an ex-ante tax on debt, $\tau_{b,0} > 0$, can enhance welfare by addressing the pecuniary externality via a reduction of debt $l_{b,0}$ that allows raising $c_{b,1}$ and thereby q_1 (see 39). This intervention can thereby implement a constrained efficient allocation as defined in Stiglitz (1982) or Davila et al. (2012), confirming findings in studies on macroprudential regulation.[23]

Proposition 3 *Consider a laissez faire equilibrium. A constrained efficient allocation can be implemented by introducing an ex-ante Pigouvian tax on debt satisfying*

$$\tau^d_{b,0} = c_{b,0} \beta z h E_0 \left[\mu^{pr}_1 r^L_1 (\partial q_1 / \partial c_{b,1}) \right] \geq 0, \tag{41}$$

where $\mu^{pr}_1 \geq 0$ denotes the multiplier on the borrowing constraint of the policy problem.

Proof. See Appendix. ■

Like in Bianchi (2011), the constraint efficient allocation under ex-ante Pigouvian debt taxes can equivalently be implemented via margin requirements. Following Bianchi's (2011) notation, margin requirements can be specified in our model by choosing a value $\theta_t \in [0, 1)$ such that the collateral constraint changes from (4) to $l_{b,t} \leq (1 - \theta_t) \cdot z q_t h_{b,t}$, which effectively leads to a contingent loan-to-value ratio $\widetilde{z}_t = (1 - \theta_t) z \leq z$. Apparently, an ex-ante margin requirement can only be effective if it leads to a binding collateral constraint in period 1. When a margin

[22] Subsequently, we will argue that margin requirements can serve as an alternative instrument.

[23] For the limiting case where the full market value of housing serves as collateral, i.e. $z \to 1$, the asset price q_1 would be equal to q^{fb}_1 even in state $s = c$, i.e. $q_1(c) = (1 + \beta) h^{-1}$ (see 39), and would be independent of consumption $c_{b,1}$. Then, the laissez faire equilibrium would be constrained efficient. Yet, this property does not in general imply that other policies, specifically, ex-post policies, cannot enhance welfare even further.

requirement is introduced such that the collateral constraint binds, it does not only reduce the debt level, but it also exerts an adverse effect on the collateral price. To see this, recall that borrowers take into account that their own housing serves as collateral. A reduction in the de-facto collaterizable fraction of housing from z to $\widetilde{z}_t$ tends to reduce the collateral premium of housing (see 5). Accordingly, the collateral price is altered by the effective loan-to-value ratio $\widetilde{z}_0$ when the borrowing constraint binds in period 0, $q_0(c_{b,0}, c_{b,1}, \widetilde{z}_0)$.[24] To implement the beginning-of-period debt level in period 1 of the constrained efficient allocation under an ex-ante Pigouvian debt tax, $l_{b,0}^{pr}$, a margin requirement θ_0 has to satisfy $l_{b,0}^{pr} = (1-\theta_0) z q_0(c_{b,0}^{pr}, c_{b,1}^{pr}, (1-\theta_0)z)h$. Thereby, margin requirements can be equivalent to an ex-ante Pigouvian debt tax.

4.4 Ex-post asset purchases

Now consider ex-post asset purchases as a policy that is applied to address inefficiencies under laissez faire (see Definition 2). Specifically, let the central bank purchase loans contingent on the state c where the borrowing constraint is binding, i.e. $\widetilde{\kappa}_1^L(c) > 0$. Then (25) and (27) imply the real loan rate $r_2^L = R_1^L \pi_2^{-1}$ to satisfy

$$\beta r_2^L(c) = 1 - [(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c) \leq 1, \tag{42}$$

instead of $\beta r_2^L(c) = 1$ (see 36). Thus, asset purchases lower the real loan rate if the purchase price exceeds the market price, $1/R_1^A(c) > 1$. Apparently, such a policy exerts effects on the loan rate that are equivalent to effects of an ex-post Pigouvian loan subsidy paid to banks. Specifically, the effects of asset purchases on the real loan rate can be mimicked by a subsidy at rate $\tau_1^s(c) \geq 0$ that affects loan supply by $(1 + \tau_1^s(c))\, \beta r_2^L(c) = 1$, and that is financed and compensated by a lump-sum tax on depositors, $\tau_{l,1}(c) = \tau_1^s(c) l_{j,1}(c)$.

Corollary 4 *Under $R_t^m = \kappa_t^B = 1$, asset purchases satisfying $1/R_1^A(c) > 1$ and $\widetilde{\kappa}_1^L(c) > 0$ and an ex-post Pigouvian loan supply subsidy are equivalent with regard to their effects on the real loan rate r_2^L and the allocation of commodities.*

Proof. See Appendix. ■

Given that $R_t^m = R_t = 1$ holds under laissez faire, monetary policy does not earn any interest from money supply or maturing assets. Thus, when assets are purchased at an above market price, transfers to the fiscal authority τ_t^m are negative (see 20). As a consequence, financing asset purchases requires funds to be raised by the fiscal authority, like in the case of loan supply

[24] Notably, this effect of margin requirements is absent in Bianchi (2011), where the borrowing limit is taken as given by borrowers (since collateral solely consists of aggregate values) and no collateral premium exists.

subsidies. The implementation of both policies therefore relies on the availability of lump-sum taxes when $R_t^m = 1$.

We now show that an asset purchase policy can enhance welfare and outperform macroprudential regulation, i.e. ex-ante debt taxes. To understand the effects of this ex-post policy, combine (5) with (7), to get the following price relation

$$q_1 = \frac{(1+\beta)\, h^{-1}}{(1-z)c_{b,1}^{-1} + z\beta r_2^L}. \tag{43}$$

The reason for the impact of the real loan rate on the asset price q_1 (see RHS of 43) is the fact that borrowers take the collateral premium of housing into account. Precisely, a lower loan rate, which raises agents' willingness to borrow and thereby the multiplier $\zeta_{b,t}$ on the collateral constraint (see 6), tends to increase the valuation of collateral (see 5). In a laissez faire equilibrium, the loan rate is exogenously fixed by $r_2^L = 1/\beta$, such that the collateral price q_1^{lf} does not seem to depend on the loan rate in (39). Yet, ex-post asset purchases can reduce the real loan rate below $1/\beta$ (see 42) and can thereby raise q_1 (see 43). Via this effect, ex-post asset purchases can address the pecuniary externality and can enhance efficiency.[25] We show that the central bank can implement a constrained efficient allocation that pareto-dominates the constrained efficient allocation under macroprudential regulation. For this, asset purchase instruments R_1^A and $\widetilde{\kappa}_1^L$ are set contingent on an equilibrium object (i.e., $c_{b,1}$), which corresponds to widely applied contingent adjustments of the monetary policy rate (e.g. the so-called Taylor-rule).

Proposition 4 *Consider a laissez faire equilibrium. Introducing ex-post asset purchases that satisfy*

$$[(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c) = \frac{1-z}{z} c_{b,1}^{-1}(c) + 1 - \frac{\alpha}{z}, \quad \text{for } \alpha < (1+\beta)\, h^{-1}/q_1^{pr}, \tag{44}$$

where q_1^{pr} denotes the collateral price under the ex-ante Pigouvian tax on debt (41), implements a constrained efficient allocation that pareto-dominates the constrained efficient allocation under the ex-ante Pigouvian tax on debt (41).

Proof. See Appendix. ■

Asset purchases satisfying (44) lead to a collateral price $q_1 = (1+\beta)h^{-1}/\alpha$ that does not depend on consumption $c_{b,1}$. Hence, there exists no pecuniary externality with regard to q_1 and the allocation is constrained efficient. For $\alpha < (1+\beta)\, h^{-1}/q_1^{pr}$, the collateral price under (44) exceeds the collateral price q_1^{pr} under the ex-ante Pigouvian tax on debt (41), such that the borrowing

[25] For the limiting case $z \to 1$, where the asset price q_1 satisfies $q_1 = (1+\beta)\, h^{-1}/(\beta r_2^L)$, asset purchases would – in contrast to an ex-ante debt tax – still have a direct impact on q_1 and would be able to enhance efficiency relative to laissez faire.

limit is increased. Since borrowers are less constrained, the allocation is pareto-superior compared to the case of the ex-ante Pigouvian tax on debt, which further distorts agents' intertemporal choice. Evidently, a sufficiently large loan rate reduction can raise the collateral price q_1 to a level that ensures that the borrowing limit exceeds the debt level under the first best allocation $l_{b,1}^{fb}$. Thereby, asset purchases implement first best. This result, which is summarized in the following proposition, corresponds to the implementation of first best by ex-post debt subsidies in Katagiri et al. (2017) and Schabert (2024).

Proposition 5 *Consider a laissez faire equilibrium. Introducing ex-post asset purchases implements the first best allocation (38) if*

$$[(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{l_{b,1}^{fb}(c)} > 0, \tag{45}$$

where $l_{b,1}^{fb}(c) = R_1^L \pi_1^{-1}(c) \left(R_{-1}\pi_0^{-1} l_{b,-1} - y_{b,0} + 1\right) - y_{b,1}(c) + 1$, *or if but not only if* $\left[(1/R_1^A(c)) - 1\right] \cdot \widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}(c)+1} > 0.$

Proof. See Appendix. ■

Notably, the RHS of condition (45) depends on the endogenous equilibrium object $l_{b,1}^{fb}(c)$, which is a function of the inflation rates $\pi_1(c)$ and π_0. Up to now, we ignored these inflation rates in the analysis to reduce the complexity of the analysis. In Section 5, we show how inflation can be determined in equilibrium and that inflation can be isolated from asset purchase effects by the remaining monetary policy instruments. In the last part of Proposition 5, we provide – as an alternative – a sufficient condition that is independent of inflation, and depends on lagged ($n_{b,0}$) or exogenous variables ($y_{b,1}(c)$).

The role of the collateral premium The previous results have shown that loan rate reductions via asset purchases (or loan supply subsidies) can enhance efficiency through their effect on the collateral price q_1. This mechanism is based on agents' willingness to spend for collateral, which is measured by the collateral premium and can be enhanced by raising their incentives to borrow, as shown by Bianchi and Mendoza (2018). Yet, if the borrowing limit were independent of the individual stock of collateral, like in Bianchi (2011) or Jeanne and Korinek (2020), this effect would not exist. If for example, the borrowing limit rather depends on the aggregate than the individual level of housing, i.e. $l_{b,1} \leq zq_1 h$, there would be no collateral premium. In this case, borrowers' optimality condition for housing satisfies $c_{b,1}^{-1} q_1 = h^{-1} + \beta q_2$, implying – together with $q_2 = h^{-1}$ – a collateral price q_1 that equals $q_1 = c_{b,1} h^{-1}(1+\beta)$ and that is independent of the real loan rate r_2^L. Changes in r_2^L then exclusively affect the multiplier $\zeta_{b,1}$ on the collateral constraint (4) via the borrowers' optimality condition $\zeta_{b,1} = c_{b,1}^{-1} - \beta r_2^L$ and the allocation of commodities between borrowers and lenders in period 2; the latter being irrelevant for social welfare due to linearity of utility in period 2.[26]

4.5 Non-availability of taxes

We now examine the case where lump-sum taxes/transfers are not available. For simplicity, we do not endogenize this property, which can for example be justified by infeasibility due to unobservable characteristics (see Hammond, 1979). Non-existence of lump-sum taxes/transfers suffices to rule out implementation of first best and of the type of Pigouvian policies (debt tax and loan subsidy) discussed above. We further abstract from introducing distortionary taxes, which would tend to reduce efficiency over and above the financial friction, and assume that there are no taxes available. As a consequence, neither corrective policies can be tax-financed nor initial liabilities can be repaid via tax revenues. As indicated by the intertemporal budget constraint (22), public sector solvency then requires that central bank revenues are raised by setting $R_t^m > 1$ in at least one period.[27] Before we examine asset purchases, we summarize how public sector solvency can be induced by monetary policy when taxes are not available.

Corollary 5 *When taxes are not available, public sector solvency can be brought about by $R_1^m > 1$, $R_0^m = R_2^m = 1$ and $R_1^A \geq 1$.*

Proof. See Appendix. ■

If $R_t^m > 1$, agents are not willing to hold deposits at the satiation level, $d_{l,t} < \overline{d}$. To avoid further complexities, we therefore assume that the marginal utility of deposits is then constant.

Assumption 3 *The marginal utility of deposits is constant, $f'(d_{l,t}) = \gamma > 0$, for $d_{l,t} < \overline{d}$.*

Under $R_1^m > 1$ and Assumption 3, the competitive equilibrium conditions (7), (25), (26), and (27) imply that $\mu_1 = \widetilde{\gamma} = \gamma/\widetilde{\mu} > 0$ and that the loan rate under ex-post asset purchases satisfies

$$\beta r_2^L(c) = 1 - \left[(\widetilde{\gamma}/R_1^A(c)) + ((1/R_1^A(c)) - 1)\right] \widetilde{\kappa}_1^L(c). \tag{46}$$

[26] A comparison of our set-up to Jeanne and Korinek (2020) shows that they consider a transfer of funds to borrowers as ex-post policies rather than a manipulation of prices, which we consider in our analysis. As a consequence, the type of ex-post policies which they apply is effective in their model even though there are no collateral premia or direct price effects of ex-post policies.

[27] Notably, asset purchases at a price $1/R_t^A < 1 \Leftrightarrow R_t^A > 1$ are not sufficient for this purpose, since banks would not sell loans at a below-market price if $R_t^m = 1$.

When the policy rate satisfies $R_1^m > 1$ such that money is scarce, $\mu_1 = \widetilde{\gamma} > 0$ (see 28), asset purchases can be effective even if the central bank offers a purchase price that is below the market price of loans, $1/R_1^A < 1$. Thus, banks might be willing to acquire money via asset purchases even if this is costly, $R_1^A > 1$.

For a constant marginal utility of deposits, we can separate the competitive equilibrium, as summarized in the Corollaries 2 and 3. Corresponding to the results in Propositions 4 and 5, it can then be shown that asset purchases can implement allocations of commodities that are identical with the constrained efficient allocations or with the first best allocation.

Lemma 1 *Consider a competitive equilibrium under Assumption 3. Suppose that taxes are not available and that $R_1^m > 1$. Then, ex-post asset purchases can implement an allocation of non-durable goods that is identical to an*

1. *allocation of non-durable goods under constrained efficiency if*

$$\left[(\widetilde{\gamma}/R_1^A(c)) + (1/R_1^A(c)) - 1\right] \cdot \widetilde{\kappa}_1^L(c) = \frac{1-z}{z} c_{b,1}^{-1}(c) + 1 - \frac{\alpha}{z}, \tag{47}$$

2. *allocation of non-durable goods under first best $\{c_{b,0}^{fb}, c_{b,1}^{fb}(s)\}$ if*

$$\left[(\widetilde{\gamma}/R_1^A(c)) + (1/R_1^A(c)) - 1\right] \cdot \widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}(c) + 1} > 0. \tag{48}$$

Proof. See Appendix. ■

Once it has been shown that asset purchases can enhance efficiency of the commodity allocation (see Lemma 1), we can use that there are sufficiently many monetary policy instruments available to implement a particular equilibrium level of deposits independent of asset purchases. Put differently, any level of deposits $d_{l,t}$ in a competitive equilibrium without asset purchases can also be implemented in a competitive equilibrium with asset purchases. For this, the money supply effect of asset purchases can be completely neutralized by adjusting the size and the price of treasury open market operations, i.e. by setting R_1^m and $\widetilde{\kappa}_1^B$. Due to the separation properties summarized in the Corollaries 2 and 3, this neutralization does not affect the allocation of commodities. Thus, the central bank can alter the collateral price by asset purchases, while keeping a particular level of deposits unchanged. Thereby, it can enhance social welfare compared to any competitive equilibrium without asset purchases.

Proposition 6 *Suppose that taxes are not available, Assumption 3 holds, and $R_1^m > 1$. Then, central bank asset purchases can enhance social welfare compared to any competitive equilibrium without asset purchases.*

Proof. See Appendix. ■

Assumption 3 and the implied separability (see Corollary 2 and 3) allow deriving the result summarized in Proposition 6 in a straightforward way. Yet, the beneficial effects of asset purchases presented above do not rely on the simplifying assumptions made in this analysis, but on the availability of a sufficiently large set of monetary policy instruments. Precisely, the latter enable the central bank to exert pure price effects by its asset purchases programmes, while neutralizing potentially welfare-reducing effects on the supply of central bank money. This property thus relates to the neutralization of the budgetary effects of distortionary taxes/subsidies via lump-sum taxes/transfers under the Pigouvian policies discussed above. Moreover, the available monetary policy instruments further allow implementing inflation targets irrespective of asset purchases, which is shown in the subsequent section.

5 Money and inflation

The previous analysis has focussed on the allocation of commodities and on social welfare. To facilitate the analysis, we imposed assumptions which allowed separating the allocation of commodities and loans from monetary variables, specifically, monetary aggregates and inflation rates (see Corollary 2 and 3). In this section, we focus on the latter, which are typically targeted by the central bank. Specifically, we will show that inflation rates are not necessarily affected by asset purchases, implying that monetary policy can be conducted such that corrective asset purchases do not interfere with common central bank targets.

As described above, there is no utility from deposit holdings and no trade in financial markets in period 2. Hence, end-of-period asset holdings equal zero, $m_2 = b_2 = d_2 = l_2 = 0$. According to (34), treasury open market operations then satisfy $I_2^B = -M_1$, such that money is redeemed by the central bank in exchange for bonds, which mature at the end of period 2. As implied by Corollary 3, the set of monetary variables $\{i_1^L, d_0, d_1, R_0, R_1, \pi_0, \pi_1, \pi_2, m_0, m_1, b_0, b_1, i_0^B, i_1^B, i_2^B\}$ can be determined for a given allocation of commodities and loans, and for policies $\{\widetilde{\kappa}_0^B, \widetilde{\kappa}_1^B, \Omega_0, \Omega_1, R_0^m, R_1^m, R_2^m, \tau_{l,1}, \tau_{l,2}, \tau_{l,3}\}$ under Assumption 1 and 2. Evidently, the central bank has several instruments at its disposal to influence the inflation rates π_0, π_1, and π_2, and to neutralize potential effects of asset purchases on the inflation rates. Following the structure of the efficiency analysis, we separately discuss the case where lump-sum taxes are available, such that first best can be implemented and deposit demand is satiated, and the case where taxes are not available and the marginal utility of deposits is constant (see Assumption 3).

In Section 4.4, we have shown how asset purchases can enhance welfare and can even implement first best when lump-sum taxes are available (see Proposition 4 and 5). For the latter, the asset purchase instruments $R_1^A(c)$ and $\widetilde{\kappa}_1^L(c)$ are assumed to satisfy (45), where the RHS depends – via $l_{b,1}^{fb}(c)$ – on the inflation rates π_0 and $\pi_1(c)$. The following proposition therefore focusses on the determination of the inflation rates π_0, π_1, and π_2 under first best, where deposits are at the satiation level. For this, we distinguish three different fiscal policy regimes: *i.*) a *Ricardian* regime that unconditionally guarantees public sector solvency, i.e. zero end-of-period public sector liabilities in $t = 2$, *ii.*) a *conditional non-Ricardian* regime where tax revenues net of costs for corrective policies are specified regardless of public sector solvency, and *iii.*) an *unconditional non-Ricardian* regime where gross tax revenues are specified regardless of public sector solvency. Thus, in addition to the well-known regimes *i.*) and *iii.*) (see e.g. Benhabib et al., 2001), we introduce regime *ii.*), to separate financing costs of corrective policies from repayment of liabilities.

Proposition 7 *Suppose that asset purchases satisfying (45) are introduced in a laissez faire equilibrium, such that first best is implemented. Then, the inflation rate in $t = 2$ satisfies $\pi_2 = \beta$, while inflation rates in $t = 0$ and $t = 1$ are i.) indetermined under a Ricardian fiscal policy regime, can ii.) be determined and are independent of asset purchases under a conditional non-Ricardian regime, and can iii.) be determined and depend on the costs of asset purchases under an unconditional non-Ricardian regime.*

Proof. See Appendix. ■

Proposition 7 establishes the determination of the initial inflation rate under asset purchases satisfying (45) with equality, which is the least expensive policy that implements first best. Corresponding to well-established results in the literature (see e.g. Nakajima and Polemarchakis, 2005), inflation determination depends on the type of fiscal policy regime. Specifically, inflation rates cannot be determined under a Ricardian regime (see *i.*)), irrespective of asset purchases, as commonly found in flexible price models. Under a non-Ricardian regime, inflation rates can be determined, since public sector solvency requires inflation to equalize the real value of initial nominal liabilities with the present value of real revenues (see 22). When tax revenues are controlled net of asset purchase costs under a conditional non-Ricardian regime, asset purchases do not affect the inflation determination. When fiscal policy is unconditionally non-Ricardian, inflation determination is affected by the costs of corrective policies. Higher costs of asset purchases, which reduce the net revenues of the public sector, then tend to increase initial inflation. Evidently, this would likewise be the case under alternative policies, i.e. Pigouvian subsidies.

For the case where taxes are not available such that fiscal policy is unconditionally non-Ricardian, we account for the neutralization of asset purchase effects on deposits, which has already been used for the derivation of the results in Proposition 6. Here, we further establish that there are sufficiently many instruments such that inflation can be isolated from asset purchases. For the analysis, we will use that monetary policy is able to generate revenues such that the public sector is solvent even though taxes are not available (see Corollary 5). For this, we again consider that the central bank imposes costs of money acquisition in period 1, $R_1^m > 1$, corresponding to Proposition 6. Then, the money supply constraint (10) is binding, $\mu_1 > 0$, which follows from (28), and deposit demand is not satiated, $u'(d_{l,1}) > 0 \Rightarrow d_{l,1} < \overline{d}$, which follows from (35). Notably, neutralization of asset purchase effects on inflation and deposits does not require using all available central bank instruments, in particular, the share of repo money Ω_t can be held constant or even be set equal to zero.

Proposition 8 *Suppose that taxes are not available, Assumption 3 holds, and $R_t^m > 1$. Then, adjustments of the treasury open market instruments $\widetilde{\kappa}_1^B(c)$, $\widetilde{\kappa}_0^B$, and R_0^m suffice to implement feasible values for deposits d_0 and d_1 and for inflation rates π_0, π_1 and π_2 independently of asset purchase programs.*

Proof. See Appendix. ■

Proposition 8 confirms that asset purchases can be conducted without affecting the equilibrium values of deposits, which has already been used for Proposition 6, and implies that asset purchases do not impede implementing targeted values for the inflation rate. Notably, this result does not depend on whether prices are perfectly or imperfectly flexible. The simple reason is that the central bank has enough instruments for treasury open market operations, i.e. $\widetilde{\kappa}_t^B$ and R_t^m, at its disposal to offset the impact of asset purchases on money supply and inflation.[28] Thus, the central bank does not need to sacrifice common objectives, e.g. regarding broad money or inflation, when it corrects asset prices via loan purchases.

6 Robustness

In this section, we address factors that might limit the efficacy and the desirability of asset purchases. Specifically, we consider arguments for potential costs of central bank asset purchases or, more generally, of ex-post liquidity-providing policy interventions, raised by Curdia and Woodford (2011), Gertler and Karadi (2011), Bornstein and Lorenzoni (2018), Jeanne and Korinek

[28] This principle is also applied in Schabert (2015) for the analysis of optimal monetary policy in a related model with sticky prices and without financial markets frictions.

(2020), Chi et al. (2024), and Amador and Bianchi (2024). To motivate these costs, we found in total six arguments/effects that are suggested to lead to distortions or deadweight losses:

1. Inferiority of the central bank to extract value from asset holdings
2. Distributive effects between borrowers and lenders
3. Raising taxes and/or issuing government debt to finance policy interventions
4. Creating credit and identifying preferred private sector investments
5. Inefficient investments at interest rates below the natural rate
6. Producing central bank money

In the subsequent analysis, we will explicitly analyze 1. and 2., which are stressed by Bornstein and Lorenzoni (2018), Jeanne and Korinek (2020), and Amador and Bianchi (2024). The reasons for the remaining arguments/effects not to be relevant for our analysis can be summarized as follows: We disregard 3., which is considered by Gertler and Karadi (2011) and Jeanne and Korinek (2020), because costs of financing corrective policies in principle apply to all policy interventions and not solely to ex-post asset purchases. In fact, this argument has explicitly been addressed in Section 4.5, where we assumed that taxes are not available and where asset purchases do not rely on government financing. Curdia and Woodford (2011) and Gertler and Karadi (2011) argue that credit creation by the central bank lead to deadweight losses (see 4.). This does not apply to our framework where the central bank rather purchases existing debt securities in secondary markets than originates loans. Because neither capital investment nor reductions in the overall level of interest rates are considered in our analysis, inefficiencies related to 5., which is mentioned by Jeanne and Korinek (2020), do not exist in our framework. Due to assets' imperfect substitutability, asset purchases change the spread between the loan rate and other interest rates (e.g. on bonds or deposits), not the level of interest rates on other assets. This property would be unchanged even when investment possibilities in productive capital were introduced. Finally, we can neglect costs of producing reserves (see 6.), which are considered by Chi et al. (2024), since our proposed asset purchase programs are constructed to be neutral with regard to the total supply of reserves and deposits (see Proposition 6).

While we focus on the analysis of asset purchases, it should be noted that the arguments/effects 2., 3., and 5. are in general also relevant for Pigouvian taxes/subsidies, which is most obvious for potential effects of taxes and of transfers that are required for compensation/financing. Likewise, Pigouvian taxes/subsidies can lead to interest rate effects and distributive effects under less restrictive assumptions on preferences (see Section 6.2).

6.1 Inferior value extraction from central bank asset holdings

When the central bank purchases assets, it supplies money under repurchase agreements against bank loans. As long as these repos are always settled, value extraction from asset holdings is not relevant. Yet, suppose that uncommitted banks repudiate the repo contract and do not repurchase loans that were purchased by the central bank at the price $1/R_t^A$. In this case, the central bank will hold loans until maturity, while the value of these loans $L_{b,t}^c$ ultimately depends on the central bank's ability to seize borrowers' collateral.

To account for an inferior ability of the central bank to extract value from their assets in a way that is consistent with our assumption on the underlying imperfection (see Section 3.1), we assume that the central bank can only seize the fraction z^c of borrowers' housing and that z^c is strictly smaller than the fraction z that banks can seize. Notably, this does not directly affect banks' loan supply decisions, which are based on the unchanged collateral requirement (4). To account for potential losses due to insufficient collateralization, the central bank can adjust the size of its asset purchase program by offering loan purchases at a haircut $1 - z^c/z$, such that the money supply restriction (9) changes to

$$I_{j,t}^L \leq \widetilde{\kappa}_t^L (z^c/z) L_{j,t}/R_t^A. \tag{49}$$

where $z^c/z \leq 1$. To achieve identical effects of central bank interventions compared to the case without inferior value extraction, the central bank can apply a suited haircut and simultaneously adjust the asset purchase instruments ($1/R_t^A$ or $\widetilde{\kappa}_t^L$), such that the total impact of asset purchases on the real loan rate is unchanged. Thus, none of the results derived in the previous sections is altered by the inferior value extraction of central bank assets.

Corollary 6 *The ability of ex-post asset purchases to enhance efficiency is not affected by an inferior value extraction from central bank asset holdings.*

Proof. See Appendix. ■

A higher purchase price, which the central bank offers to compensate haircuts, leads to higher budgetary costs of asset purchase programs. When lump-sum taxes are available, this effect will not be relevant for any other equilibrium object. When lump-sum taxes are not available, a higher purchases price can be financed via increased central bank interest earnings by raising the policy rate R_t^m, which is irrelevant for the allocation of commodities and social welfare in our model (see Corollary 2).

6.2 Distributive effects

We now assess if distributive effects might alter the effectiveness and desirability of ex-post interventions. For the previous analysis, we assumed that utility of depositors/lenders is linear in consumption as well as utility of borrowers in period 2. As a consequence, distributive effects of changes in the loan rate r_2^L induced by asset purchases were not relevant for social welfare. Here, we change the assumptions on preferences and assume that utility is always non-linear in consumption. To simplify the analysis, we disregard uncertainty and assume that borrowers will be constrained in period 1 with certainty. This is summarized in the following assumption, which replaces Assumption 1 and 2.

Assumption 4 *Agents' preferences satisfy*

$$u_{b,t} = \log(c_{b,t}) + \log(h_{b,t}), \text{ for } t \in (0,1,2),$$
$$u_{l,t} = \log(c_{l,t}) + f(d_{l,t}), \text{ for } t \in (0,1), \text{ and } u_{l,2} = \log c_{l,2},$$

with $f_d(\overline{d}) = 0$ *and* $f_d(d_{l,t}) > 0$ *if* $d_{l,t} > \overline{d}$. *There are no net taxes/transfers on/to borrowers, and the borrowing constraint is slack in* $t = 0$ *and 2, and binds in* $t = 1$.

As shown in Section 4.4, a reduction in the real loan rate r_2^L due to asset purchases reduces the rate of return on lending and therefore the costs of debt repayment. Thus, asset purchases tend to induce a redistribution of resources in period 2 from lenders/depositors to borrowers (see 23). Under linear utility in period 2, this redistribution was irrelevant for social welfare. This irrelevance does however not apply under Assumption 4, which might alter the effectiveness and desirability of asset purchases. Moreover, welfare weights cannot be normalized to equal one under Assumption 4 without loss of generality. We therefore assume that social welfare W satisfies $W = E\sum_{t=0}^{T}\beta^t\left(\phi_b u_{b,t} + \phi_l u_{l,t}\right)$, with $\phi_i > 0$ for $i \in \{b,l\}$, instead of (37).

Yet, even if the loan rate r_2^L affects social welfare via redistribution of funds in period 2, asset purchases are desirable. The reason is that borrowers suffer from being constrained in period 1 and are characterized by a higher marginal utility of consumption. The latter gives rise to distributive effects of pecuniary externalities with regard to the loan rate, which becomes endogenous under Assumption 4. As shown by Davila and Korinek (2020), these distributive effects arise under different marginal rates of substitution due to binding borrowing constraints. Ideally, corrective policies should be applied in a way that addresses both effects of externalities, i.e. collateral effects and distributive effects. Given that both are based on binding borrowing constraints and an asset purchase program can in fact be applied to induce the borrowing constraint to be slack (see Proposition 5 and Lemma 1), it can address the joint source of adverse effects.

Proposition 7 *Suppose that Assumption 4 holds and that either lump-sum taxes/transfers are available or that Assumption 3 holds. Then, an optimal asset purchase policy reduces the loan rate such that the collateral constraint is not binding.*

Proof. See Appendix. ■

The result summarized in Proposition 7 implies that an asset purchase policy that leads to slack collateral constraints remains desirable even when distributive effects are considered. Specifically, unequal marginal rates of substitution between borrowers and lenders are then eliminated by asset purchases, such that distributive effects due to changes in the real loan rate are irrelevant for social welfare.

7 Conclusion

Financial stability has been threatened by the great financial crisis as well as by the Covid-19 pandemic, where central banks intervened at a large scale and seemed to exert beneficial effects on asset prices. This raises the question if (ex-post) monetary policy might be superior to (ex-ante) macroprudential regulation. This paper develops a monetary model with a financial amplification mechanism based on pecuniary externalities, where macroprudential regulation implements a constrained efficient allocation. We show that central bank asset purchases in secondary markets can implement constrained efficient allocations that pareto-dominate the constrained efficient allocation under macroprudential regulation and can even implement first best. While these effects can equivalently be induced by a Pigouvian loan supply subsidy, we show that the central bank can conduct welfare-enhancing asset purchases even when taxes are not available for financing/compensating policy interventions. This property is based on the central bank's ability to raise revenues via interest earnings from money supply and asset holdings. We further show that asset purchases do not impede achieving (conventional) central bank targets/objectives, and that neither distributive effects nor inferior value extraction from central bank asset holdings invalidate the conclusions regarding the desirablity of asset purchases.

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A Appendix to Section 3

Banks' reserve demand This appendix aims at providing a rationale for the banks' liquidity constraint (10), for which we acknowledge that reserves are required for the settlement of deposit transactions of banks (see Bianchi and Bigio, 2022): Suppose that a bank i faces an idiosyncratic shock $\omega_{i,t}$ that determines the (net) volume of deposits that have to be sent to other banks. A bank who has to make payments draws $\omega_{i,t} > 0$, and a receiving bank draws $\omega_{i,t} < 0$, while cross sectional expectation satisfies $E_i \omega_{i,t} = 0$. Suppose that the shock $\omega_{i,t}$ is i.i.d. distributed with $\omega_{i,t} \in (\omega^{\min}, \omega^{\max}]$, and $\omega^{\max} \leq 1$. Settlement of deposit transactions in a gross settlement system (like Fedwire) requires an equally sized amount of reserves, such that the maximum amount of reserves required by bank i is $\omega^{\max} D_{i,t}$. For this, the bank can hold reserves from the previous period $M_{i,t-1}$ and they can get new reserves from treasury open market operations $I_{i,t}^{B}$ or asset purchase programs $I_{i,t}^{L}$, which are offered before idiosyncratic shocks are realized.

Suppose that banks can further borrow (or lend) reserves in an interbank market, which opens after idiosyncratic shocks are realized. Let $F_{i,t} > 0$ ($F_{i,t} < 0$) denote the amount borrowed (lent) by bank i in the interbank market, with $\int F_{i,t} di = 0$. Thus, the demand for reserves is satisfied by $\omega_{i,t} D_{i,t} \leq F_{i,t} + M_{i,t-1} + I_{i,t}^{B} + I_{i,t}^{I}$. Given that interbank markets are typically over-the-counter, interbank transactions rely on search and matching (see e.g. Bianchi and Bigio, 2022). Let γ^{IB} be the matching probability of interbank offers, such that the probability of unmatched offers is $1 - \gamma^{IB}$. Taking the latter and maximum reserve demand into account, bank i's precautionary demand for reserves can be summarized as $(1 - \gamma^{IB}) \omega^{\max} D_{j,t} \leq M_{i,t-1} + I_{i,t}$. Hence, the fraction $\widetilde{\mu}$ in (10) can be rationalized by the probability of an interbank market mismatch times the maximum requirement for deposit transactions, $\widetilde{\mu} = (1 - \gamma^{IB}) \omega^{\max}$.

Proof of Corollary 1. Suppose that money is supplied under full allotment in treasury open market operations $\widetilde{\kappa}_t^B = 1$ at the price $1/R_t^m = 1$ and that no asset purchases are offered, $\widetilde{\kappa}_t^L = 0$. According to (28), the multipliers on the liquidity constraint (10) and on the money supply constraint (8) are then identical, $\mu_t = \kappa_t^B$, while (18) implies that the bond rate satisfies $R_t = 1$. Hence, holdings of public sector liabilities are neither associated with costs (in the case of money) nor with positive interest earnings (in the case of bonds). Given that asset purchases are not offered, $\widetilde{\kappa}_t^L = 0$, credit supply (25) equals $u'(c_{l,t}) = \beta R_t^L E_t u'(c_{l,t+1}) \pi_{t+1}^{-1}$. Now suppose that the multiplier μ_t were positive, $\mu_t > 0$. Then, (26) would imply that the deposit rate is strictly lower than the loan rate, such that banks generate positive profits by supplying loans and issuing deposits. Given that banks are perfectly competitive, profits (11) are however equal to zero, implying that $R_t^L = R_t^D$ and that μ_t must be equal to zero. Deposit demand is then satiated (see 35), $u'(d_{l,t}) = 0 \Rightarrow d_{l,t} = \overline{d}$. For $\mu_t = 0$, (30) further simplifies to $u'(c_{l,t}) = \beta E_t u'(c_{l,t+1})\,\pi_{t+1}^{-1}$, such that net interest rates on loans and deposits are equal to zero, $R_t^L = R_t^D = 1$. ■

Proof of Corollary 2. It follows from Definition 1 and from (7) being independent of $d_{l,t}$ if $\partial u'(d_{l,t})/\partial d_{l,t} = 0$. ■

Proof of Corollary 3. It follows from Definition 1. ■

B Appendix to Section 4

Proof of Proposition 1. Under Assumption 1, maximizing (37) subject to the resource constraints $\Sigma_i y_{i,t} = \Sigma_i c_{i,t}$ implies that $c_{b,0}^{fb} = c_{b,1}^{fb}(s) = c_{l,1}^{fb}(s) = 1$, and that $d_{l,0} = d_{l,1} = \overline{d}$. In a laissez faire equilibrium, $\mu_{j,t} = 0$ and $d_{l,0} = d_{l,1} = \overline{d}$ hold (see Definition 2). When borrowing is never constrained, $\zeta_{b,t} = 0$, the competitive equilibrium conditions (5), (6), and (25) for $t = 0, 1,$ and 2 simplify to $c_{b,1}^{-1} q_1(s) = h^{-1} + \beta q_2$, $q_2 = h^{-1}$, $c_{b,0}^{-1} = \beta E_0[r_1^L(s) c_{b,1}^{-1}(s)]$, $c_{b,1}^{-1}(s) = \beta r_2^L$, and (36), implying that $c_{b,0}^{-1} = 1$, and $c_{b,1}^{-1}(s) = 1$ for $s \in \{c, u\}$. Hence, the allocation equals first best and $q_1(s) = (1+\beta)h^{-1}$ for $s \in \{c, u\}$. ■

Proof of Proposition 2. In a laissez faire equilibrium (see Definition 2) under Assumptions 1 and 2, borrowers' credit demand (6) and banks' credit supply (25) for $t = 0$ and $t = 1$ satisfy (36), $c_{b,0}^{-1} = \beta E_0[r_1^L(s) c_{b,1}^{-1}(s)]$, $c_{b,1}^{-1}(s) = \beta r_2^L + \zeta_{b,1}(s)$, and therefore $c_{b,1}^{-1}(u) = 1$ and $c_{b,1}^{-1}(c) = 1 + \zeta_{b,1}(c) > 1$, since $\zeta_{b,1}(c) > 0$. Substituting out $c_{b,1}^{-1}(u)$ and $c_{b,1}^{-1}(c)$ in $c_{b,0}^{-1} = \beta 0.5[r_1^L(u) \cdot c_{b,1}^{-1}(u) + r_1^L(c) \cdot c_{b,1}^{-1}(c)]$, shows that $c_{b,0}^{-1} = \beta 0.5\left[r_1^L(u) \cdot 1 + r_1^L(c) \cdot \left(1 + \zeta_{b,1}(c)\right)\right] = \beta E r_1^L + \beta 0.5 r_1^L(c)\zeta_{b,1}(c) > 1$ and thus $c_{b,0} < 1$. The borrowers' optimality condition for housing (5) further implies $c_{b,1}^{-1}(s) q_1(s) = h^{-1} + \beta q_2 + \zeta_{b,1}(s) z q_1(s)$ and $q_2 = h^{-1}$, such that $q_1(u) = c_{b,1}(u)\,(1+\beta)\,h^{-1}$ and $q_1(c) = (1+\beta)\,h^{-1}/(c_{b,1}^{-1}(c) - \zeta_{b,1}(s)z)$. Substituting out $\zeta_{b,1}(c)$ with $c_{b,1}^{-1}(c) = 1 + \zeta_{b,1}(c)$ in the latter, gives $q_1(c) = (1+\beta)\,h^{-1}/[(1-z)c_{b,1}^{-1}(c) + z]$, which is strictly smaller than $q_1^{fb}(s) = h^{-1}(1+\beta)$, since $c_{b,1}(c) < 1$. ■

Proof of Proposition 3. Under laissez faire, W (see 37) can be rewritten by using $h_{b,t} = h$, lenders' budget constraints, and that agents are satiated with deposits, $d_{l,t} = \overline{d}$:

$$W = E\left\{\begin{array}{c} \log c_{b,0} + \log(h) + \left(y_{l,0} + r_0^L l_{b,-1} - l_{b,0}\right) + f(\overline{d}) \\ +\beta\left[\log c_{b,1} + \log(h) + y_{l,1} - l_{b,1} + r_1^L l_{b,0} + f(\overline{d})\right] + \beta^2\left[y_{b,2} + \log(h) + y_{l,2} + f(\overline{d})\right] \end{array}\right\}.$$

The primal problem of a policy maker who applies an ex-ante tax on debt $\tau_{b,0}$ and a compensating lump-sum transfer is identical to the problem of a social planer who determines period-0-borrowing and maximizes social welfare W subject to budget and borrowing constraints taking the equilibrium price relations (36) and (39) into account, leading to a constrained efficient allocation. The policy problem can be summarized as

$$\max_{c_{b,1},c_{b,2},l_{b,1},l_{b,2}} E\{\log c_{b,0} + \log(h) + \left(y_{l,0} + r_0^L l_{b,-1} - l_{b,0}\right) + f(\overline{d}) \quad (50)$$

$$+ \beta \left[\log c_{b,1} + \log(h) + y_{l,1} - l_{b,1} + r_1^L l_{b,0} + f(\overline{d})\right] + \beta^2 \left[y_{b,2} + \log(h) + y_{l,2} + f(\overline{d})\right]\}$$

$$\text{s.t.} \quad 0 = y_{b,0} + l_{b,0} - r_0^L l_{b,-1} - c_{b,0},\ 0 = y_{b,1} + l_{b,1} - r_1^L l_{b,0} - c_{b,1},\ 0 \leq zq_1 h - l_{b,1},$$

where $q_1 = q_1^{lf}$ satisfies $q_1^{lf}(u) = q_1^{fb}$ and (39), leading to the optimality conditions $\lambda_{b,0}^{pr} = 1/c_{b,0}$,

$$\lambda_{b,1}^{pr} = (1/c_{b,1}) + \mu_1^{pr} zh\partial q_1/\partial c_{b,1}, \quad (51)$$

$$1 = \beta E_0 r_1^L + \lambda_{b,0}^{pr} - \beta E_0(r_1^L \lambda_{b,1}^{pr}), \quad (52)$$

$$\mu_1^{pr} = \beta(\lambda_{b,1}^{pr} - 1) \geq 0, \quad (53)$$

where $\lambda_{b,0}^{pr}$, $\lambda_{b,1}^{pr}$, and μ_1^{pr} are the multipliers for the constraints in order of their appearance in (50). Using $E_0 r_1^L = 1/\beta$ (see 36), condition (52) simplifies to $\lambda_{b,0}^{pr} = \beta E_0 r_1^L \lambda_{b,1}^{pr}$. Multiplying (51) with βr_1^L, applying expectations conditional on period-0-information $E_0 \beta r_1^L \lambda_{b,1}^{pr} = E_0 \left(\beta r_1^L / c_{b,1}\right) + E_0[\beta r_1^L \mu_1^{pr} zh(\partial q_1/\partial c_{b,1})]$, and using $1/c_{b,0} = \lambda_{b,0}^{pr} = \beta E_0 r_1^L \lambda_{b,1}^{pr}$ as well as (40), gives

$$1/c_{b,1} = (1 - \tau_{b,0}^d) c_{b,1}^{-1} + \beta E_0[r_1^L \mu_1^{pr} zh(\partial q_1/\partial c_{b,1})], \quad (54)$$

leading to condition (41) for the tax rate on debt, where the inequality in (41) follows from $\partial q_1(c)/\partial c_{b,1} > 0$ and $\mu_1^{pr}(c) \geq 0$ (see 39 and 53). ■

Proof of Corollary 4. According to Definition 1, ex-post asset purchases $\widetilde{\kappa}_1^L(c) > 0$ can affect the equilibrium allocation via credit supply (25), which simplifies to (42), and money supply (32). According to Corollary 2, monetary aggregates do not affect the allocation of commodities. Hence, asset purchases can alter the latter only via changes in the real loan rate (see 42). This effect on the real loan rate can equivalently be induced by an ex-post subsidy on loans, satisfying $\tau_1^s(c) = \{1 - [(1/R_1^A(c)) - 1]\widetilde{\kappa}_1^L(c)\}^{-1} - 1 > 0$, which tends to increase bank profits and leads to a loan supply of banks satisfying $(1 + \tau_1^s(c))\, \beta r_2^L(c) = 1$, and by compensating the bank owners (depositors) by an equally sized lump-sum tax $\tau_{l,1}(c) = \tau_1^s(c) l_{j,1}(c)$. ■

Proof of Proposition 4. As shown in the proof of Proposition 3, a constrained efficient allocation satisfies

$$1/c_{b,0} = E_0\left(\beta r_1^L/c_{b,1}\right) + E_0[\beta r_1^L \mu_1^{ce} zh(\partial q_1/\partial c_{b,1})]. \tag{55}$$

(see 40 and 54). Now consider the price relation (43) instead of (39) and suppose that $s = c$. The impact of consumption $c_{b,1}$ on q_1, which is not internalized by private agents, can be offset if r_2^L ensures that $\partial q_1/\partial c_{b,1} = 0$. For this, suppose that asset purchases satisfy (44). Combining the latter with (42), gives $\beta r_2^L(c) = \frac{\alpha}{z} - \frac{1-z}{z} c_{b,1}^{-1}(c)$, and substituting out βr_2^L in (43), leads to

$$q_1(c) = \frac{(1+\beta)\, h^{-1}}{\alpha}. \tag{56}$$

Since $\partial q_1/\partial c_{b,1} = 0$ (see 56), condition (55) reduces to $1/c_{b,0} = E_0\left(\beta r_1^L/c_{b,1}\right)$. Given that the latter is identical with the optimality condition for borrowing under laissez faire, the competitive equilibrium allocation under an asset purchase policy (44) is constrained efficient. For $\alpha < (1+\beta)\, h^{-1}/q_1^{pr}$, the asset price $q_1(c)$ exceeds the asset price under the ex-ante Pigouvian debt tax $q_1^{pr}(c)$, such that the borrowing limit is increased. Given that lenders are unaffected and borrowers are less constrained, the allocation under an asset purchase policy (44) pareto-dominates the allocation under the ex-ante Pigouvian debt tax (41). ■

Proof of Proposition 5. Under the first best allocation, borrowing in $t = 1$ satisfies

$$l_{b,1}^{fb}(s) = r_1^L(s)\left(r_0^L l_{b,-1} - y_{b,0} + 1\right) - y_{b,1}(s) + 1, \tag{57}$$

where we used $y_{b,0} = -l_{b,0} + r_0^L l_{b,-1} + c_{b,0}$, $y_{b,1} = -l_{b,1} + r_1^L l_{b,0} + c_{b,1}$, and (38). Since borrowing is unconstrained under first best, implementation of a first best allocation in a competitive equilibrium requires the asset price q_1 to satisfy $q_1(c) \geq l_{b,1}^{fb}(c)/(zh)$, such that the borrowing constraint (4) is slack. Using that $q_1(c)$ satisfies (43) in a competitive equilibrium, this inequality can be rewritten as

$$\frac{z\,(1+\beta)}{(1-z) + z\beta r_2^L(c)} \geq l_{b,1}^{fb}(c), \tag{58}$$

where we used that $c_{b,1}^{fb} = 1$. Using condition (42), asset purchases satisfying (45) ensure that the inequality (58) holds. Alternatively, asset purchases can be conditioned on lagged or exogenous variables. For this, use (57) for $s = c$, and add $r_1^L(u)\left(r_0^L l_{b,-1} - y_{b,0} + 1\right) = r_1^L(u) l_{b,0}^{fb} > 0$ on the RHS of (57), to get

$$\begin{aligned} l_{b,1}^{fb}(c) &< \left(r_0^L l_{b,-1} - y_{b,0} + 1\right)\left(r_1^L(c) + r_1^L(u)\right) - y_{b,1}(c) + 1 \\ &= (1 - n_{b,0})\, 2\beta^{-1} - y_{b,1}(c) + 1 \end{aligned}$$

where $n_{b,0} = y_{b,0} - r_0^L l_{b,-1}$ and we used $E_0 r_1^L = 0.5\left(r_1^L(c) + r_1^L(u)\right) = 1/\beta$. The borrowing constraint will therefore not be binding, i.e. (58) holds, if but not only if the real loan rate satisfies $\frac{z(1+\beta)}{(1-z)+z\beta r_2^L(c)} \geq (1-n_{b,0})\, 2\beta^{-1} - y_{b,1}(c) + 1 \Leftrightarrow$

$$\beta r_2^L(c) \leq \frac{(1+\beta)}{(1-n_{b,0})\, 2\beta^{-1} - y_{b,1}(c) + 1} - \frac{1-z}{z}.$$

Using (42), the latter inequality is ensured by setting the instruments $1/R_1^A$ and $\widetilde{\kappa}_1^L$ according to $\left[(1/R_1^A(c)) - 1\right] \widetilde{\kappa}_1^L(c) \geq \frac{1}{z} - \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}(c)+1} > 0$. ■

Proof of Corollary 5. When taxes are not available and ex-post asset purchases are available for $t = 1$, the intertemporal public sector budget constraint (22) for $T = 2$ reduces to

$$\begin{aligned} b_{-1}\pi_0^{-1} + R_0^m m_{-1}\pi_0^{-1} = & \left(R_0^m - (R_1^m/R_0) + (R_0^m - 1)\,\Omega_0\right) m_0 \\ & + (R_1^m - (R_2^m/R_1) + (R_1^m - 1)\,\Omega_1) m_1\, (\pi_1/R_0) \\ & + \left(R_1^A - 1\right) i_1^L\, (\pi_1/R_0)\,. \end{aligned} \tag{59}$$

Now use that (28) and (29) imply $1/R_0 = \beta E_0 (1 + \widetilde{\kappa}_1^B\, ((1+\mu_1)\,(1/R_1^m) - 1)))\pi_1^{-1}$. Together with $1 = \beta E_0\,(1+\mu_1)\,\pi_1^{-1}$ (see (30)), this indicates that R_0 equals R_1^m for $\widetilde{\kappa}_1^B = 1$ and for a non-state-contingent R_1^m, and tends to be larger for $\widetilde{\kappa}_1^B < 1$. Since there are no treasury open market operations in $t = 2$, $\widetilde{\kappa}_2^B = 0$, (29) implies $1/R_1 = \beta\pi_2^{-1}$, while (30) and μ_2 imply $\pi_2 = \beta$ and thus $R_1 = 1$. Hence, with $R_1^m(s) > 1$ for $s \in (c,u)$ the term $R_0^m - (R_1^m/R_0)$ cannot be negative for $R_0^m = 1$ and $R_1^m - (R_2^m/R_1)$ is strictly positive for $R_2^m = 1$, such that the RHS of (59) is strictly positive irrespective of Ω_0 and Ω_1. For given values $b_{-1} > 0$ and $m_{-1} > 0$, (59) is solved by $\pi_0 > 0$. ■

Proof of Lemma 1. Suppose that $R_1^m > 1$, such that (28) and (35) imply $u'(d_{l,1}) > 0$, with $u'(d_{l,1}) = \gamma$ under Assumption 3. Given that the marginal utility of deposits is constant, consumption, debt, and the asset price do not depend on deposits and on monetary policy instruments, except of asset purchases, as summarized in Corollary 2. Analogously to the case of satiated deposit demand (see Proposition 4), asset purchases can then implement the allocation of non-durable goods under constrained efficiency by ensuring that the real loan rate for $s = c$ satisfies $\beta r_2^L = \frac{1-z}{z} c_{b,1}^{-1} - \frac{\alpha}{z}$. It follows directly from (46) that this requires asset purchases under $R_1^m > 1$ to satisfy (47). Correspondingly, asset purchases can implement the allocation of non-durable goods under first best by ensuring that the real loan rate for $s = c$ satisfies $\beta r_2^L \leq \frac{1+\beta}{(1-n_{b,0})2\beta^{-1} - y_{b,1}+1} - \frac{1-z}{z}$ (see Proposition 5). It directly follows from (46) that the latter inequality requires asset purchases to satisfy (48). ■

Proof of Proposition 6. Suppose that Assumption 3 holds and $R_1^m > 1$, $R_0^m = R_2^m = 1$ and $R_1^A \geq 1$, such that solvency (22) is ensured. Then, (28) and (35) imply that the liquidity constraint (33) is binding, $\mu_1 > 0$, where $\mu_1 = \widetilde{\gamma} = \gamma/\widetilde{\mu}$ (see (35)). Assume that the central bank sets R_1^m and R_1^A according to $R_1^m \in (1, 1+\widetilde{\gamma})$ and $R_1^A \in (1, 1+\widetilde{\gamma})$. Then, (27) and (28) imply $\kappa_1^L > 0$ and $\kappa_1^B > 0$, such that (31) and (32) are binding and the liquidity constraint (33) can be written as

$$\widetilde{\mu} d_1 = \widetilde{\kappa}_1^B \left(b_0 \pi_1^{-1} / R_1^m \right) + \widetilde{\kappa}_1^L \left(l_1 / R_1^A \right) + m_0 \pi_1^{-1}. \tag{60}$$

Condition (60) determines d_1 for a set of equilibrium values for π_1, b_0, l_1, m_0, and monetary policy instruments R_1^A, $\widetilde{\kappa}_1^L$, R_1^m and $\widetilde{\kappa}_1^B$, where the last two instruments are irrelevant for the allocation of non-durables and housing (see Corollary 2). Further use that social welfare W (37) can be separated as $W = W_{c,h} + W_d$, where $W_{c,h} = \log c_{b,0} + c_{l,0} + \beta E \left[\log c_{b,1} + c_{l,1} \right] + \beta^2 y + (1 + \beta + \beta^2) \log(h)$ and $W_d = \gamma d_{l,0} + \beta E \gamma d_{l,1}$. Now use that asset purchases can enhance efficiency of the non-durable goods allocation according to Lemma 1 and can thus yield a higher value $W_{c,h}$ compared to a competitive equilibrium without asset purchases, $\widetilde{\kappa}_1^L = 0$. According to (60), the impact of asset purchases, i.e. $\widetilde{\kappa}_1^L (l_1 / R_1^A)$, and potentially of π_1 on the equilibrium value for deposit d_1 can be offset by adjustments of $\widetilde{\kappa}_1^B$ and R_1^m, such that W_d is unaffected by asset purchases and social welfare W is strictly larger than without asset purchases. ■

C Appendix to Section 5

Proof of Proposition 7. When asset purchases satisfying (45) are introduced in a laissez faire equilibrium, where $R_t^m = \widetilde{\kappa}_t^B = 1$, $\mu = 0$, and $d_1 = d_2 = \overline{d}$, first best is implemented (see Proposition 5). Then, the money supply constraints (31) and (32), and the liquidity constraint (33) are slack, such that i_0^B, i_1^B, m_0 and m_1 are indetermined. According to (25) and (30), the loan rates R_0^L and R_1^L and the inflation rates π_1 and π_2 then satisfy $R_0^L = 1$, $R_1^L(c) = 1 - (1/R_1^A(c) - 1)\widetilde{\kappa}_1^L(c)$, $R_1^L(u) = 1$, $1/\beta = E_0 \pi_1^{-1} \Leftrightarrow$

$$1/\beta = 0.5(\pi_1^{-1}(c) + \pi_1^{-1}(u)), \tag{61}$$

and $\pi_2 = \beta$. Using the terminal conditions, $m_2 = b_2 = 0$, absence of asset purchases in $t = 0$ and $t = 2$, $i_0^L = i_2^L = 0$, and that $R_t^m = 1$ and $\widetilde{\kappa}_t^B = 1$ imply $R_t = 1$ (see 18), the public sector budget constraints (21) for $t = 0, 1$, and 2 reduce to

$$b_0 + \tau_{l,0} + \left(m_0 - m_{-1}\pi_0^{-1}\right) = b_{-1}\pi_0^{-1},$$
$$b_1 + \tau_{l,1} + \left(m_1 - m_0\pi_1^{-1}\right) + \left(R_1^A - 1\right) i_1^L = b_0\pi_1^{-1}, \quad \tau_{l,2} + \left(-m_1\pi_2^{-1}\right) = b_1\pi_2^{-1},$$

which can – by substituting out b_0 and b_1 – be integrated to get

$$(m_{-1} + b_{-1})\,\pi_0^{-1}\pi_1^{-1} = \tau_{l,0}\pi_1^{-1} + \tau_{l,1} + \tau_{l,2}\beta - \left((1/R_1^A) - 1\right)\widetilde{\kappa}_1^L l_{b,1}, \tag{62}$$

where we used that (32) is binding, $i_1^L = \widetilde{\kappa}_1^L l_{b,1}/R_1^A$, under (45). Given that (62) holds for all states s and that asset purchases in $t = 1$ are state contingent, we apply (62) separately for both states $s = c$ and $s = u$. For $s = c$, we suppose that (45) holds with equality. Using the latter and that loans under first best in $s = c$ satisfy $l_{b,1}^{fb}(c) = R_0^L\pi_1^{-1}(c)\left(R_{-1}^L\pi_0^{-1}l_{b,-1} - y_{b,0} + 1\right) + 1 - y_{b,1}(c)$, we can rewrite (62) for $s = c$ as follows

$$\pi_1^{-1}(c) = \frac{\tau_{l,1}(c) + \tau_{l,2}\beta + (1+\beta) - \frac{1}{z}\left(1 - y_{b,1}(c)\right)}{(m_{-1} + b_{-1})\,\pi_0^{-1} + \left[\frac{1}{z}\left(R_{-1}^L\pi_0^{-1}l_{b,-1} - y_{b,0} + 1\right)\right] - \tau_{l,0}}, \tag{63}$$

where we used $R_0^L = 1$. Likewise, we apply (62) for $s = u$, and rewrite it as follows

$$\pi_1^{-1}(u) = \frac{\tau_{l,1}(u) + \tau_{l,2}\beta}{(m_{-1} + b_{-1})\,\pi_0^{-1} - \tau_{l,0}}, \tag{64}$$

where we used $i_1^L(u) = 0$. We now consider three fiscal policy regimes: Under a Ricardian regime, where zero public sector liabilities $m_2 = b_2 = 0$ are unconditionally guaranteed by tax revenues, (62) does not constitute an additional restriction, such that π_0, $\pi_1(c)$ and $\pi_1(u)$ cannot be determined. On the contrary, (62) imposes a restriction on the inflation rates if the fiscal policy regime is non-Ricardian, i.e. tax revenues do not guarantee zero end-of-period public sector liabilities in period 2. Under a conditionally non-Ricardian regime, taxes are adjusted to cover asset purchase costs $\left((1/R_1^A) - 1\right)\widetilde{\kappa}_1^L l_{b,1}$. Let taxes $\tau_{l,0}$, $\widetilde{\tau_{l,1}}$ and $\tau_{l,2}$ be non-Ricardian, where $\widetilde{\tau_{l,1}}$ are period-1-taxes $\tau_{l,1}$ net of asset purchase costs. Then, (62) and (63) can be written as $(m_{-1} + b_{-1})\,\pi_0^{-1}\pi_1^{-1} = \tau_{l,0}\pi_1^{-1} + \widetilde{\tau_{l,1}} + \tau_{l,2}\beta$ and

$$\pi_1^{-1}(c) = \frac{\widetilde{\tau_{l,1}}(c) + \tau_{l,2}\beta}{(m_{-1} + b_{-1})\,\pi_0^{-1} - \tau_{l,0}}. \tag{65}$$

Using (61), the inflation rates π_0, $\pi_1(c)$ and $\pi_1(u)$ are then determined by

$$\pi_0 = \frac{m_{-1} + b_{-1}}{\tau_{l,0} + 0.5\beta\left(\widetilde{\tau_{l,1}}(c) + \tau_{l,1}(u)\right) + \beta^2\tau_{l,2}},$$

(64) and (65), and are independent of asset purchases. Now suppose that taxes $\tau_{l,0}$, $\tau_{l,1}$ and $\tau_{l,2}$ are unconditionally non-Ricardian, irrespective of asset purchase costs. Using (61), the inflation rates π_0, $\pi_1(c)$ and $\pi_1(u)$ can then be determined by

$$2\beta^{-1} = \frac{\tau_{l,1}(u) + \tau_{l,2}\beta}{(m_{-1} + b_{-1})\,\pi_0^{-1} - \tau_{l,0}} + \frac{\tau_{l,1}(c) + \tau_{l,2}\beta + (1+\beta) + \frac{1}{z}\,(1 - y_{b,1}(c))}{\left(m_{-1} + b_{-1} + \frac{1}{z}R_{-1}^{L}l_{b,-1}\right)\pi_0^{-1} + \frac{1}{z}\,(1 - y_{b,0}) - \tau_{l,0}},$$

(63) and (64), and evidently depend on the costs of asset purchases. ■

Proof of Proposition 8. Suppose that taxes are not available and Assumption 3 holds. Under Assumption 1, (30) implies $\pi_2 = \beta$. Suppose further that $R_0^m \in (1, 1+\widetilde{\gamma})$ and $R_1^m \in (1, 1+\widetilde{\gamma})$, such that $\mu_0 = \mu_1 = \widetilde{\gamma} > 0$ (see 7 and 26) and that the money supply constraint (31) binds in $t = 0$ and $t = 1$ (see 28). Then, substitute out i_0^B and i_1^B with (34) in the binding money supply constraint (31) and in the binding liquidity constraint (33), to get for $t = 0$ and $t = 1$

$$\widetilde{\kappa}_0^B b_{-1}\pi_0^{-1}/R_0^m = (1+\Omega_0)m_0 - m_{-1}\pi_0^{-1}, \quad \widetilde{\kappa}_1^B b_0\pi_1^{-1}/R_1^m = (1+\Omega_1)m_1 - m_0\pi_1^{-1}, \qquad (66)$$

$$\widetilde{\mu}d_0 = (1+\Omega_0)\,m_0, \quad \widetilde{\mu}d_1 = (1+\Omega_1)\,m_1 + i_1^L. \qquad (67)$$

Without taxes, the consolidated public sector budget constraint (21) reduces to $(b_t/R_t) + R_t^m(m_t - m_{t-1}\pi_t^{-1}) + (R_t^m - 1)\,\Omega_t m_t + \left(R_t^A - 1\right) i_t^L = b_{t-1}\pi_t^{-1}$. Substituting out b_0 and b_1 with the latter for $t = 1$ and $t = 2$, $b_0\pi_1^{-1} = (b_1/R_1) + R_1^m\left(m_1 - m_0\pi_1^{-1}\right) + (R_1^m - 1)\,\Omega_1 m_1 + \left(R_1^A - 1\right) i_1^L$ and $-m_1 R_2^m = b_1$, leads to the following versions of the integrated public sector budget constraint and the money supply condition (66) for $t = 1$

$$\begin{aligned} b_{-1}\pi_0^{-1} + R_0^m m_{-1}\pi_0^{-1} = {} & (R_0^m - (R_1^m/R_0) + (R_0^m - 1)\,\Omega_0)\,m_0 \\ & + (R_1^m - (R_2^m/R_1) + (R_1^m - 1)\,\Omega_1)m_1\,(\pi_1/R_0) \\ & + \left(R_0^A - 1\right) i_0^L + \left(R_1^A - 1\right) i_1^L\,(\pi_1/R_0)\,, \end{aligned} \qquad (68)$$

$$\begin{aligned} \left((1+\Omega_1)m_1 - m_0\pi_1^{-1}\right) R_1^m/\widetilde{\kappa}_1^B = {} & (-\,(R_2^m/R_1) + R_1^m + (R_1^m - 1)\,\Omega_1)\,m_1 \\ & - R_1^m m_0\pi_1^{-1} + \left(R_t^A - 1\right) i_t^L, \end{aligned} \qquad (69)$$

while (28), (29), and (30) imply $1/R_0$ to satisfy

$$1/R_0 = \beta E\left(1 + \widetilde{\kappa}_1^B\left((1+\widetilde{\gamma})\,(1/R_1^m) - 1\right)\right)\pi_1^{-1}. \qquad (70)$$

To simplify the analysis, we further restrict the choice of policy instruments. Firstly, we set $R_2^m = R_1 = 1$, using that R_1 satisfies $1/R_1 = \beta\pi_2^{-1}$ (see 29) and $\pi_2 = \beta$ (see 30). Secondly, we set the asset purchase price at $R_1^A = R_1^m \in (1, 1+\widetilde{\gamma})$, such that (32) binds in $t = 1$ (see

27). Thirdly, we assume that $\Omega_0 = 0$. Using $i_0^L = 0$, $i_1^L = \widetilde{\kappa}_1^L l_{b,1}/R_1^A$, and the binding liquidity constraints (67) to substitute out i_1^L, m_0 and m_1 in (68), (69), and in the money supply condition (66) for $t = 0$, leads to

$$b_{-1}\pi_0^{-1} + R_0^m \left(m_{-1}\pi_0^{-1} - \widetilde{\mu} d_0\right) = (R_1^m/R_0)\widetilde{\mu}\left(d_1\pi_1 - d_0\right) - \widetilde{\mu} d_1\left(\pi_1/R_0\right), \tag{71}$$

$$(1/R_1^m) = ((1/\widetilde{\kappa}_1^B) - 1)\left([d_0/\left(d_1\pi_1\right)] - 1\right) + \widetilde{\kappa}_1^L l_{b,1}/(R_1^A\widetilde{\kappa}_1^B), \tag{72}$$

$$\widetilde{\kappa}_0^B b_{-1}\pi_0^{-1}/R_0^m = \widetilde{\mu} d_0 - m_{-1}\pi_0^{-1}. \tag{73}$$

Hence, the set of equilibrium values $\{\pi_0, \pi_1, d_0, d_1\}$ has to satisfy (71), (72), (73), and

$$[(1+\widetilde{\gamma})\beta]^{-1} = E\pi_1^{-1}, \tag{74}$$

with R_0 satisfying (70), for a given set of monetary policy instruments $\{R_0^m$, $R_1^m = R_1^A$, $\widetilde{\kappa}_1^L(s)$, $\widetilde{\kappa}_0^B$, $\widetilde{\kappa}_1^B\}$ and an equilibrium value for loans $l_{b,1}$. Suppose that the central bank targets specific values for deposits, $d_0 = \widehat{d}_0$ and $d_1 = \widehat{d}_1$, and for the inflation rate, $\pi_0 = \widehat{\pi}_0$ and $\pi_1 = \widehat{\pi}_1$ within the range of feasible values, implying $\pi_1 = (1+\widetilde{\gamma})\beta$ (see 74), and sets its instruments accordingly. In a competitive equilibrium without asset purchases, $\widetilde{\kappa}_1^L = 0$, condition (72) determines a set of associated values for the pair R_1^m and $\widetilde{\kappa}_1^B$. Given that both are non-state-contingent, R_0 satisfies $1/R_0 = \frac{1-\widetilde{\kappa}_1^B}{1+\widetilde{\gamma}} + \frac{\widetilde{\kappa}_1^B}{R_1^m}$ (see 70). Thus, a particular pair $\overline{R}_1^m$ and $\overline{\widetilde{\kappa}}_1^B$ determine a value for $\overline{R}_0^m$ according to (71), while the latter leads to a particular value for $\overline{\widetilde{\kappa}}_0^B$ according to (73).

Now suppose that the central bank introduces asset purchases in state $s = c$ in period 1, $\widetilde{\kappa}_1^L(c) > 0$ and $\widetilde{\kappa}_1^L(u) = 0$. For an unchanged value $\overline{R}_1^m$, (72) implies a higher value for $\widetilde{\kappa}_1^B(c)$ than $\overline{\widetilde{\kappa}}_1^B$, leading to a lower bond rate R_0 according to (70). Given that $\overline{R}_1^m > 1$, the RHS of (71) is decreasing in R_0. Hence, (71) implies a higher value for R_0^m than $\overline{R}_0^m$, and (73) a higher value for $\widetilde{\kappa}_0^B$ than $\overline{\widetilde{\kappa}}_0^B$. To summarize, an ex-post asset purchase policy $\widetilde{\kappa}_1^L(c) > 0$ with $R_1^A(c) = R_1^m$ are fully neutralized with regard to the level of real deposits and the inflation rate by raising $\widetilde{\kappa}_1^B(c)$, $\widetilde{\kappa}_0^B$, and R_0^m. ■

D Appendix to Section 6

Proof of Corollary 6. Suppose that the central bank can only seize a fraction z^c of borrowers' collateral and adjusts asset purchase programs by introducing haircuts $1 - z^c/z$, such that the money supply restriction (9) changes to (49). Then, replacing the asset purchase instrument $\widetilde{\kappa}_1^L$ by $\widehat{\kappa}_1^L = \widetilde{\kappa}_t^L(z^c/z)$, in (45), (44), (47), and (48), leaves all efficiency results summarized in the Propositions 4, 5 and 8 unaffected. ■

Proof of Proposition 7. Under Assumption 4, the following optimality conditions hold

$$c_{b,0}^{-1} = \beta r_1^L c_{b,1}^{-1}, \tag{75}$$

$$c_{l,0}^{-1} = \beta r_1^L c_{l,1}^{-1}, \tag{76}$$

$$c_{l,1}^{-1} = \beta r_2^L c_{l,2}^{-1} + \kappa_1^L \widetilde{\kappa}_1^L / R_1^A, \tag{77}$$

$$\kappa_1^L = \mu_1 - c_{l,1}^{-1}(R_1^A - 1), \tag{78}$$

$$c_{b,1}^{-1} = \beta r_2^L c_{b,2}^{-1} + \zeta_{b,1}, \tag{79}$$

$$c_{b,1}^{-1} q_1 = u'(h) + \beta c_{b,2}^{-1} q_2 + \{\zeta_{b,1} z q_1\}. \tag{80}$$

Substituting out κ_1^L and $\zeta_{b,1}$ with (78) and (79) in (77) and (80), leads to

$$c_{l,1}^{-1} = \beta r_2^L c_{l,2}^{-1} + \left(u'(d_{l,1})\,\widetilde{\mu}^{-1} - c_{l,1}^{-1}(R_1^A - 1)\right) \widetilde{\kappa}_1^L / R_1^A, \tag{81}$$

$$q_1 = \frac{(1+\beta)u'(h)}{c_{b,1}^{-1}(1-z) + z\beta r_2^L c_{b,2}^{-1}}, \tag{82}$$

where we used $u'(d_{l,t})\,\widetilde{\mu}^{-1} = \mu_1$. Combining (75) and (76), further gives $c_{l,0}^{-1}/c_{l,1}^{-1} = c_{b,0}^{-1}/c_{b,1}^{-1}$. Using $c_{l,t} = y - c_{b,t}$, then leads to $\frac{y-c_{b,1}}{y-c_{b,0}} = \frac{c_{b,1}}{c_{b,0}}$ and thus

$$c_{b,0} = c_{b,1} \text{ and } r_1^L = \beta^{-1}. \tag{83}$$

If lump-sum taxes/transfers are available, a welfare-maximizing policy would implement the satiation level of deposits $\overline{d}$. If not and Assumption 3 holds, any equilibrium level of deposits can be implemented irrespective of asset purchases (see Corollary 3). In either case, the allocation of deposits and housing is independent of asset purchases, such that an optimal asset purchase policy can be identified by maximizing $W = E\sum_{t=0}^{2}\beta^t(\phi_b\log(c_{b,t}) + \phi_l\log(c_{l,t}))$. Using (83) and that condition (81) does not impose a constraint to the primal policy problem, the latter can be summarized as follows

$$\max_{c_{b,1},c_{b,2},l_{b,0},l_{b,1},r_2^L} \{(1+\beta)\left[\phi_b \log c_{b,1} + \phi_l \log(y - c_{b,1})\right] + \beta^2\left[\phi_b \log(c_{b,2}) + \phi_l \log(y - c_{b,2})\right]\} \tag{84}$$

$$\text{s.t.}\quad 0 = y_{b,0} + l_{b,0} - r_0^L l_{b,-1} - c_{b,1},\; 0 = y_{b,1} + l_{b,1} - \beta^{-1} l_{b,0} - c_{b,1},\; 0 = y_{b,2} - r_2^L l_{b,1} - c_{b,2},$$

$$0 \leq z q_1(c_{b,1}, c_{b,2}, r_2^L)h - l_{b,1},$$

where $q_1(c_{b,1}, c_{b,2}, r_2^L)$ satisfies (82). The first order conditions with respect to $c_{b,1}$, $c_{b,2}$, $l_{b,0}$, $l_{b,1}$

and r_2^L are

$$\lambda_{b,0}^{\text{lg}} = (1+\beta)\left(\phi_b c_{b,1}^{-1} - \phi_l (y - c_{b,1})^{-1}\right) - \beta\lambda_{b,1}^{\text{lg}} + \mu_1^{\text{lg}} zh\partial q_1/\partial c_{b,1}, \tag{85}$$

$$\lambda_{b,2}^{\text{lg}} = \phi_b c_{b,2}^{-1} - \phi_l (y - c_{b,2})^{-1} + \beta^{-2}\mu_1^{\text{lg}} zh\partial q_1/\partial c_{b,2}, \tag{86}$$

$$\lambda_{b,0}^{\text{lg}} = \lambda_{b,1}^{\text{lg}}, \qquad \lambda_{b,1}^{\text{lg}} = \beta\lambda_{b,2}^{\text{lg}} r_2^L + \mu_1^{\text{lg}}, \tag{87}$$

$$\lambda_{b,2}^{\text{lg}} l_{b,1} = \beta^{-2}\mu_1^{\text{lg}} zh\partial q_1/\partial r_2^L, \tag{88}$$

where $\lambda_{b,0}^{\text{lg}} \geq 0$, $\lambda_{b,1}^{\text{lg}} \geq 0$, $\lambda_{b,2}^{\text{lg}} \geq 0$, and $\mu_1^{\text{lg}} \geq 0$ are the multipliers for the constraints in order of their appearance in (84). Since $\partial q_1/\partial r_2^L < 0$ (see 82) and $\lambda_{b,2}^{\text{lg}}$ as well as μ_1^{lg} are non-negative, condition (88) requires, $\lambda_{b,2}^{\text{lg}} = \mu_1^{\text{lg}} = 0$. Thus, the real loan rate has to be lowered by asset purchases according to (81) such that the collateral constraint is not binding for the social planer. Then, the conditions in (87) require $\lambda_{b,1}^{\text{lg}} = \lambda_{b,0}^{\text{lg}} = 0$, such that (85) and (86) imply that the marginal utilities of consumption of borrowers and lenders are equated. ■

E Appendix to a textbook-style model

Suppose that money is supplied at $R_t^m = R_t^A = 1$, the central bank sets R_t, and that the money supply constraints (8) and (9) are not internalized by banks. Bank $i's$ liquidity constraint and profits are then given by

$$\widetilde{\mu} D_{j,t} \leq M_{j,t}, \tag{89}$$

and $P_t\omega_{j,t} = D_{j,t} - R_{t-1}^D D_{j,t-1} - L_{j,t} + R_{t-1}^L L_{j,t-1} - B_{j,t}/R_t + B_{j,t-1} - M_{j,t} + M_{j,t-1}$, such that the first order conditions for loans, bonds and money satisfy

$$\lambda_{j,t} = \beta E_t R_t^L \lambda_{j,t+1}\pi_{t+1}^{-1}, \tag{90}$$

$$\lambda_{j,t}/R_t = \beta E_t \lambda_{j,t+1}\pi_{t+1}^{-1}, \tag{91}$$

$$\lambda_{j,t} = \beta E_t \pi_{t+1}^{-1}\lambda_{j,t+1} + \mu_{j,t}, \tag{92}$$

instead of (12), (16), and (17), whereas the first order condition for deposits again satisfies (13). Combining the latter with (90)-(92), leads to

$$R_t^L = R_t, \qquad R_t^D = R_t - (R_t - 1)\widetilde{\mu}, \qquad \mu_{j,t} = \lambda_{j,t}(1 - 1/R_t),$$

where $\mu_{j,t}$ is the multiplier on (89). Under $R_t^m = R_t^A = 1$, the intertemporal budget constraint of the integrated public sector (22) further simplifies to

$$(b_{-1} + m_{-1})/\pi_0 = \sum_{t=0}^{T-1} \left(\prod_{k=1}^{t} \frac{\pi_k}{R_{k-1}} \right) \left\{ \tau_{l,t} + \tau_{b,t} + \frac{R_t - 1}{R_t} m_t \right\}. \tag{93}$$

In this version, central bank asset purchases are neutral.

Corollary 8 *Suppose that banks do not internalize (8) and (9), and that the central bank controls R_t, while $R_t^m = R_t^A = 1$. Then, purchases of loans with central bank money is neutral with regard the equilibrium allocation of commodities and deposits, such that social welfare is unchanged.*

Proof. According to the optimal behavior of banks, demand for central bank money is solely restricted by (92) or $\mu_{j,t} = \lambda_{j,t}(1 - 1/R_t)$. Given that banks are indifferent between holdings loans or bonds (see $R_t^L = R_t$), they are also indifferent between acquiring central bank money via an exchange of bonds or loans against money. Moreover, loan purchases leave asset prices unchanged (see (90)-(91)), such that they do not alter the private sector behavior compared to the case where no loans (and only bonds) are purchased by the central bank (see Definition 1). Hence, the equilibrium allocation of commodities and deposits, and therefore social welfare is not affected by loan purchases. ■

F Appendix to interest on reserves

Suppose that the central bank pays interest on reserves R_t^R (IOR). Then, bank $j's$ profits satisfy

$$\begin{aligned} P_t\omega_{j,t} = {} & D_{j,t} - R_{t-1}^D D_{j,t-1} - L_{j,t} + R_{t-1}^L L_{j,t-1} - B_{j,t}/R_t + B_{j,t-1} - M_{j,t} + R_t^R M_{j,t-1} \\ & - I_{j,t}^B \left(R_t^m - R_t^R\right) - I_{j,t}^L (R_t^A - R_t^R), \end{aligned}$$

and the affected first order conditions for reserves from treasury open market operations and from asset purchases as well as for holdings of money satisfy

$$\mu_{j,t} = \kappa_{j,t}^B + \lambda_{j,t}(R_t^m - R_t^R), \tag{94}$$

$$\kappa_{j,t}^L = \mu_{j,t} - \lambda_{j,t}(R_t^A - R_t^R), \tag{95}$$

$$\lambda_{j,t} = \beta E_t \pi_{t+1}^{-1} \left(R_{t+1}^R \lambda_{j,t+1} + \mu_{j,t+1}\right), \tag{96}$$

instead of (14), (15), and (17). Substituting out $\mu_{j,t+1}$ in (96) with (94), gives $\lambda_{j,t} = \beta E_t \pi_{t+1}^{-1}(\kappa_{j,t}^B + \lambda_{j,t+1} R_{t+1}^m)$ and thus leads to (18) for $\widetilde{\kappa}_t^B = 1$, like in the case without IOR. With IOR, (94) and (95) further indicate that costs of money acquisition from treasury open market operations and asset purchase programs depend on the interest rate spreads $R_t^m - R_t^R$ and $R_t^A - R_t^R$. Thus, reserves are abundantly available if $R_t^m = R_t^R$, not if $R_t^m = 1$ (see Corollary 1). Substituting out $\kappa_{j,t}^L$, the unchanged loan supply condition (12) together with (95) further shows that effectiveness of asset purchases under satiated money demand relies on $R_t^A < R_t^R$ instead of $R_t^A < 1$; the latter being applied for the conditions (44) and (45). The conditions on asset purchases in Propositions 4 and 5 and in Lemma 1 change accordingly, i.e. the term $(1/R_1^A(c)) - 1$ in (44), (45), (47), and (48) is replaced by $(1/R_1^A(c)) - (1/R_1^R(c))$. The integrated public sector budget constraint is – without interest payments in the terminal period ($R_T^R = 1$) – given by

$$\begin{aligned}&(b_{-1} + R_0^m m_{-1})/\pi_0 - \sum_{t=0}^{T}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)(\tau_{l,t} + \tau_{b,t}) \qquad (97)\\ &= \sum_{t=0}^{T-1}\left(\prod_{k=1}^{t}\frac{\pi_k}{R_{k-1}}\right)\left\{\begin{array}{c}\left[R_t^m - R_{t+1}^m/R_t\right]m_t + \left[(R_t^m - 1)\,\Omega_t m_t\right] + \left(R_t^A - 1\right)i_t^L \\ -\left(R_t^R - 1\right)\left(m_{t-1}\pi_0^{-1} + \Omega_t m_t + i_t^L\right)\end{array}\right\},\end{aligned}$$

instead of (22). The second line in the curly brackets on the RHS of (97) summarizes the costs of IOR, and implies higher tax revenues or central bank earnings for repayment of initial liabilities. Accordingly, the restrictions on monetary policy instruments when taxes are not available change, while the properties summarized in Corollary 5 and Proposition 8 are unaffected.