Liquidity Premia and Interest Rate Parity[1]

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Abstract

Due to the US dollar's dominant role for international trade and finance, risk-free assets denominated in US currency not only offer a pecuniary return, but also provide transactions services, both nationally and internationally. Accordingly, the responses of bilateral US dollar exchange rates to interest rate shocks should differ substantially with respect to the (US or foreign) origin of the shock. We demonstrate this empirically and apply a model of liquidity premia on US treasuries originating from monetary policy implementation. The liquidity premium leads to a modification of uncovered interest rate parity (UIP), which enables the model to explain an appreciation of the dollar subsequent to an increase in US interest rates if foreign interest rates follow the US monetary policy rate.

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1 Introduction

We study the role of liquidity premia on assets for exchange rate responses to changes in monetary policy rates. Our starting point consists of two observations. First, standard open economy macro models typically involve a version of uncovered interest rate parity (UIP), which states that the expected rate of depreciation is equal to the differential between home and foreign short-term interest rates. However, it is well established that this theoretical prediction is rarely confirmed by empirical data (see Froot and Thaler, 1992, or Engel, 2013, for surveys on the evidence). Second, returns on certain types of assets can be affected by the existence of liquidity premia. At least short-term US treasuries arguably help to facilitate market transactions, for example through their use as collateral, and the liquidity services these assets provide are non-pecuniary benefits that are reflected in their price (see e.g. Longstaff, 2004, or Krishnamurthy and Vissing-Jorgensen, 2012). The point we make in this paper results from combining these two observations. We argue that the failure of the UIP prediction, i.e. the observed lack of a positive association between interest rate differentials and expected depreciation rates, may be partly due to movements in endogenous liquidity premia.

Specifically, this paper contributes to explaining empirical evidence on short-run exchange rate dynamics in response to monetary policy shocks, i.e. the association of a home interest rate increase with a subsequent exchange rate appreciation, which has been reported in a number of empirical studies. Most notably, Eichenbaum and Evans (1995) have presented VAR evidence pointing out that a contractionary U.S. monetary policy shock leads the dollar to appreciate for many periods, until it peaks after around three years, which they summarize as delayed overshooting. More recently, Scholl and Uhlig (2008) reconfirm the result and find that the exchange rate peaks between 17 and 26 months after a monetary shock. These findings are in contrast to UIP predictions, and as such have proven difficult to explain (see Engel, 2013). This paper presents a theoretical approach to explain these findings, which is based on the specific role of the US dollar in international transactions and the implied liquidity value of US assets (specifically, treasury securities). When the US monetary policy rate increases, the price of money in terms of the collateral required in open market operations (typically treasury securities) also increases, such that the liquidity value of treasuries falls. Accounting for both the pecuniary and the non-pecuniary components of the total returns leads to an uncovered interest rate parity condition that is modified by a liquidity premium on US assets. If a higher US monetary policy rate is then followed by an increase in the foreign interest rate such that the net effect on the interest rate differential is reduced, the adverse response of the liquidity premium can lead to a subsequent exchange rate appreciation, much as found in the empirical studies quoted above.[3]

[3]In related literature, it has been noted that there may be other ways in which modelling the liquidity value of bonds may help with international empirical puzzles, such as the exchange rate volatility puzzle or the Backus-Smith puzzle, as demonstrated by Canzoneri et al. (2013a). Engel (2012) and Canzoneri et al. (2013b) also discuss how

We develop a two country model, consisting of a large domestic open economy (the US) and a small foreign open economy, where agents of both countries assign a liquidity value to US treasury securities. While several theoretical approaches have emphasized that assets other than money may be valued for their transactions services (see e.g. Lahiri and Vegh, 2003, Canzoneri et al., 2008, Linnemann and Schabert, 2010), the precise way in which assets provide liquidity is left open. In contrast, we derive the liquidity value of treasuries from the property that they are eligible in open market operations, and can thus be transformed into central bank money at a cost which equals the monetary policy rate.[4] As an implication, returns on treasuries and on non-eligible assets differ by an endogenous liquidity premium that varies with the stance of monetary policy. We recognize the fact that the US dollar has a special role in the international payments system, in that large parts of trade are conducted in this currency.[5] Since assets that can easily be liquidated in US currency are therefore particularly valuable for their holders in comparison to assets denoted in other currencies that are less important in international trade, it follows that changes in the US monetary policy rate are predicted to have different consequences than changes in interest rates of any small open economy. We show analytically (for a simplified version of the model) that a shock to the large country policy rate leads to a subsequent appreciation of its currency if the foreign interest rate responds positively to a sufficiently large extent. Then, the net effect on the interest rate differential can be dominated by the adverse change in the liquidity premium, implying an exchange rate response consistent with the empirically observed delayed overshooting. Hence, the combined effect of the policy rate on foreign interest rates and on the liquidity premium can qualitatively explain the observed exchange rate response. We confirm this result applying a calibrated version of the model, where the foreign interest rate response is endogenous and governed by a standard interest rate rule.

We assess the empirical validity of the model's implications by means of a panel vector autoregression with monthly data from the US and a number of small and medium sized open economies. We find that – in line with earlier empirical evidence – an increase in the US monetary policy rate leads to a prolonged period of appreciation. This violation of the UIP prediction is compatible with our model, as we find that the average small open economy interest rate has a peak response of about one half of the peak increase of the US rate (while there is no comparable reaction of US rates to interest rate shocks in small open economies), which is a precondition for the liquidity premium to dominate the exchange rate response according to our model. We further find that an increase in the monetary policy rate in the average small open economy triggers a response

transaction services of bonds can contribute to international empirical puzzles.

[4] The model is based on the closed economy model of Reynard and Schabert (2013), where the liquidity premium is shown to behave according to Krishnamurthy and Vissing-Jorgensen's (2012) evidence and to be able to explain the observed systematic spread between (real) monetary policy rates and the marginal rate of intertemporal substitution (see Canzoneri et al. 2007 and Atkeson and Kehoe 2009).

[5] This has been labelled as key currency pricing by Canzoneri et al. (2013a), who analyze costs and benefits of this particular status of the US dollar.

of bilateral exchange rates with respect to the dollar that is roughly in line with the UIP prediction, in that it produces an almost immediate increase in the depreciation rate of the small open economy currency against the dollar.[6] Applying our theoretical model, it is possible to explain both an appreciation of the US dollar subsequent to increases in the US monetary policy rate and in non-US interest rates. According to the model, the liquidity premium together with the empirically observed positive international linkage of interest rates is able to account for the US dollar appreciation following an increase in US interest rates relative to foreign interest rates. The model further predicts that an increase in the policy rate in a small open economy leads to an exchange rate response in accordance with UIP, which is consistent with our empirical finding.

The rest of the paper is organized as follows. Section 2 presents empirical evidence supporting the view that bilateral exchange rates between the US dollar and the currencies of small open economies deviate from the prediction of UIP when US monetary policy shocks are considered, while UIP is roughly compatible with the responses to interest rate shocks originating from small open economies. Section 3 presents the model, whereupon Section 4 analytically derives the main result for a simplified model version. The quantitative properties of a calibrated version of the model are discussed in Section 5; Section 6 concludes.

2 Empirical evidence

In this section, we present empirical evidence on asymmetries in the exchange rate responses to monetary policy shocks of different origins, which are suggestive for different roles of internationally traded assets and currencies, specifically, for the US dollar, in explaining deviations from UIP. In particular, we follow Eichenbaum and Evans (1995) and estimate VAR models to assess the impact of monetary policy shocks on exchange rates. In contrast to previous studies that either focus on US monetary policy shocks or on interest rate differentials (see Eichenbaum and Evans, 1995, Scholl and Uhlig, 2008), we distinguish between shocks to US monetary policy and shocks to monetary policy in a number of other open economies for which comparable data are available. We show that, using recursively identified VARs with monthly data, there is a pronounced and prolonged appreciation subsequent to a US policy rate shock, as already emphasized in previous literature (Eichenbaum and Evans, 1995, Scholl and Uhlig, 2008). However, we find that this behavior of exchange rates is much less pronounced in response to a shock to the monetary policy rate of open economies other than the US. The latter finding is similar to the results reported by Bjornland (2009) in her study using quarterly data of four small open economies (Australia, Canada, New Zealand, Sweden). Specifically, we find an unexpected increase in the US short-run nominal interest rate to be followed by several months of a decreasing US dollar exchange rate

[6] The empirical finding that there is hardly any deviation from UIP predictions when considering small open economies (whose currencies are not prominent in international trade) is compatible with Bjornland (2009), who confirmed that depreciation of the small open economy currency against the US dollar follows a domestic interest rate increase for Australia, Canada, New Zealand and Sweden.

(hence an appreciation), while an unexpected increase to money market interest rates in small open economies that do not share the US dollar's special role in international finance leads to an almost immediate appreciation of the local currency followed quickly by a depreciation, consistent with UIP. We further find that an increase in the US policy rate is followed by a substantial increase in small open economies interest rates (but not vice versa), implying that the interest rate differential moves by much less than the original US policy rate change.

We use monthly data to estimate a panel VAR model capturing the average bilateral interaction between the US and a number of small open economies (SOE, henceforth, see country list below). Data are mostly from the IMF's International Financial Statistics database, with few exceptions mentioned below. The VAR is estimated in the vector of variables $Z_t = [x_t^{US}, \pi_t^{US}, R_t^{m,US}, R_t^{US}, x_t^i, \pi_t^i, R_t^i, S_t^i]'$. The superscript US denotes US variables, whereas the superscript i refers to one out of the group of small open economies for which all data are available at monthly frequency. The variable x_t is the growth rate of industrial production, π_t is the CPI inflation rate, $R_t^{m,US}$ is the short-run nominal policy rate, taken to be the Federal Funds Rate for the US, R_t^{US} is the US three months treasury bill rate. The variable R_t^i is the money market interest rate in the i-th open economy outside the US, taken as a measure for the monetary policy rate. S_t^i is the log of the nominal bilateral exchange rate between the i-th country and the US dollar (denoted such that a decrease indicates a nominal appreciation of the US dollar). A monetary policy shock is an innovation to the orthogonalized residual of the equations for the interest rates $R_t^{m,US}$ and R_t^i, respectively. Identification is achieved as in Eichenbaum and Evans (1995) by assuming a contemporaneous recursive ordering where the variables are ordered as given in the definition of Z_t. This entails the assumption that US monetary policy can react contemporaneously to innovations in US production growth and inflation, but interest rate shocks affect the former two variables only after a lag of at least one month. Likewise, the central bank of the i-th small open economy is able to react to innovations in both domestic and US production and inflation, while there is a one month lag before SOE interest rate shocks affect these. The nominal exchange rate, being ordered last, can react contemporaneously to all shocks.[7]

Monthly data on seasonally adjusted industrial production, consumer price indices, the money market interest rate, and the bilateral exchange rate with respect to the US dollar are obtained from the IMF International Financial Statistics database. The full group of non-US open economies for which all data are available consists of Austria, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Japan, the Netherlands, Norway, Portugal, Spain, the UK, and the Euro area. The data series mostly begin in 1975m1 except for Finland, where data availability starts in 1978, and Portugal, where it starts in 1983m1. For Germany, the CPI data were merged pre- and post-

[7] Bjornland (2009) obtains (for a more limited data set) similar exchange rate responses by using a long-run restriction approach for the identification of monetary shocks (in quarterly data) by imposing a zero long-run effect of monetary policy on the real exchange rate.

unification by using the pre-unification inflation rates of West Germany. In a few cases, the IMF data had to be complemented from data from the OECD's Main Economic Indicator database. This pertains to the money market interest rate for Norway, and the CPI for the UK (where the IMF data is missing partly and the OECD measure was used). For the countries that joined the Euro area in 1999, the series end in 1998m12, whereas for the remaining countries the data end in 20013m12.[8] Time series for the Euro area as a whole naturally start from 1999m1. We determine the lag length using the Schwarz information criterion. For all two country pairs, this criterion suggests either using one or two lags. We consequently use a two lag specification for the VAR, though we checked that using only one or up to six lags would not lead to different conclusions. Furthermore, entering the price level and industrial production variables in log-levels, instead of growth rates, produces very similar results.

The purpose of the analysis is to establish empirically whether the exchange rate response to monetary shocks is different for a country with a currency that plays a less important role for international transactions compared to the US dollar. The latter is presumably the case for truly small open economies like (in our sample) Austria, Belgium, Canada, Denmark, Finland, Italy, the Netherlands, Norway, Portugal, and Spain. However, for medium sized countries like Germany, Japan, France, the UK, and the Euro area, the characterization as a small open economy is certainly less suited.[9] We therefore first run the panel VAR only with the countries uncontroversially belonging to the small country group, and examine the effect of additionally entering the aforementioned medium sized country group in a second step further below (results for the sample consisting solely of the medium sized countries are shown in appendix A).

For each sample, we estimate the model in the form of a non-balanced panel VAR with country fixed effects (as in Ravn, Schmitt-Grohe and Uribe, 2012, the sample size is large enough to allow us to neglect the possible source of bias from correlation between fixed effects and regressors as identified by Nickell, 1981). Also, we checked that estimating individual two country VARs and averaging the results instead of using a panel approach lead to very similar conclusions. We compute the impulse responses of the nominal exchange rate with respect to an orthogonalized positive one unit shock to the US nominal policy interest rate $R_t^{m,US}$ (indicating a monetary policy shock in the US) and with respect to an orthogonalized positive one unit shock to the nominal interest rate R_t^i (indicating a monetary policy shock in the average SOE).

In Figure 1, we show two sets of impulse responses for the VAR with the small country group, along with bootstrapped two standard deviation bands, namely in each row those of the US policy interest rate $R_t^{m,US}$, the SOE policy rate R_t^i, and the bilateral exchange rate S_t^i. The first row of panels shows responses to a unit shock to the US nominal policy interest rate, and the second row of

[8] In appendix A, we ascertain that the results remain largely the same if we exclude the Great Recession period and cut the sample off in 2007m12.

[9] We are grateful to two anonymous referees for independently pointing this out.

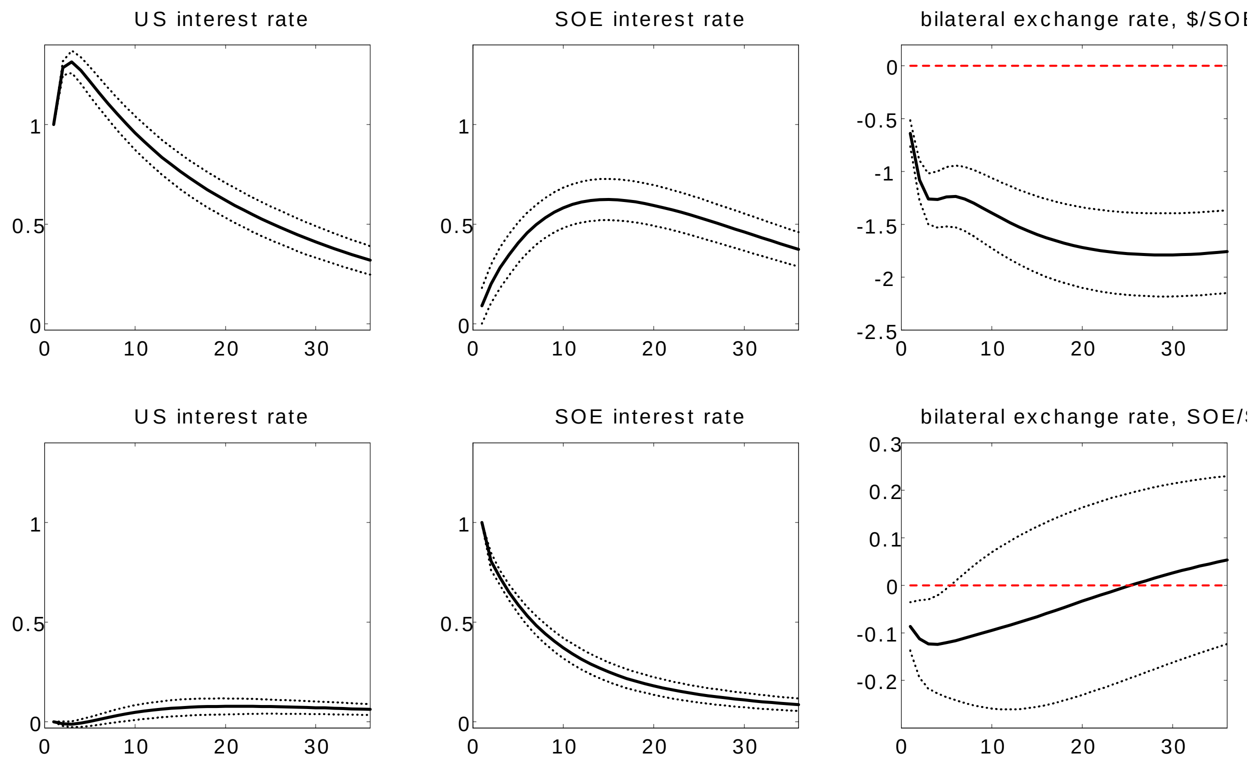

Figure 1: Impulse responses to US monetary policy shock (first row) and to small economy monetary policy shock (second row), dashed lines are two standard error confidence bands.

panels those to a shock to the SOE interest rate. For better readability, in the figure the exchange rate responses are presented from the point of view of the country in which the monetary policy shock occurs. Thus, a decrease of the exchange rate following a US monetary policy shock (upper right panel) means an appreciation of the US dollar with respect to the small open economy's exchange rate, whereas a decrease of the exchange rate following a SOE monetary policy shock (lower right panel) means an appreciation of the small economy's currency vis-a-vis the US dollar.

The upper row of panels in Figure 1 displays a result that is well known from previous studies: in response to a US monetary tightening, the US policy rate $R_t^{m,US}$ that is shown in the upper left panel increases persistently (as does the US treasury bill rate, which reacts very similarly as shown in the appendix A), and the US dollar appreciates (relatively to the SOE currencies in the sample) with a pronounced hump-shaped pattern with a peak response that occurs almost 30 months after the shock (upper right panel). This continuing appreciation for around two years after an interest rate increase is a clear violation of uncovered interest rate parity, and accords to previous findings in the literature (see Eichenbaum and Evans, 1995, Scholl and Uhlig, 2008). As can be seen from the middle panel in the first row, the SOE interest rate reacts strongly positively to a US monetary tightening. The increase in the SOE interest rate is less than one for one, such that the spread

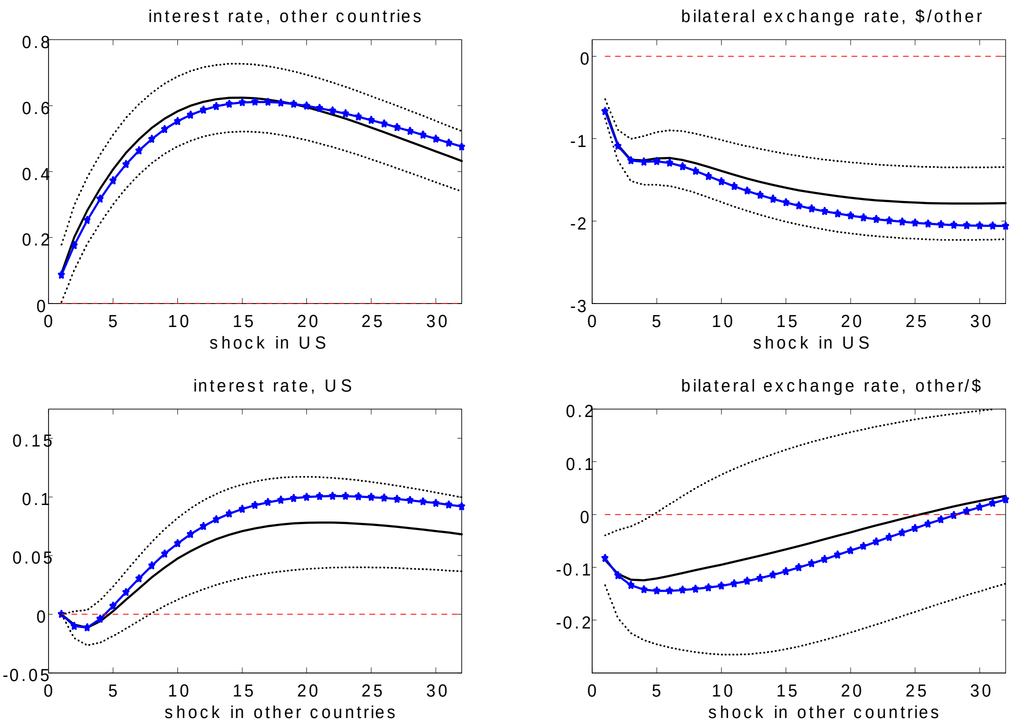

Figure 2: Impulse responses to US monetary policy shock (first row) and to monetary policy shock in small or medium sized countries (second row). Starred lines: all countries, black lines: small countries only.

between the US federal funds rate and the policy interest rate in the SOE increases. Hence, from standard UIP reasoning one would expect an immediate decline in the exchange rate followed by a subsequent depreciation of the US dollar, and thus an upward sloping response in the upper right panel, the opposite of which actually occurs.

On the other hand, looking at the second row of panels in Figure 1 shows that the UIP predictions are roughly compatible with the responses that follow a SOE monetary policy shock. An increase in the SOE nominal interest rate (lower middle panel) spurs only a very limited reaction of less than one tenth of a percentage point of the US interest rate, and is thus almost equal to the spread between both rates. As the lower right panel shows, this leads almost immediately (roughly after three months) to a subsequent depreciation. This exchange rate behavior is, from about three months after a shock onward, compatible with the prediction of UIP.

To summarize, two main results can be taken from this analysis. First, we find that exchange rate responses subsequent to US interest rate changes are inconsistent with standard UIP, whereas there is a response qualitatively in accordance with UIP for the exchange rate response to SOE interest rate changes. Second, an increase in the US interest rate due to a monetary policy tightening leads to a substantial increase in the SOE nominal interest rate. Measured at the peaks of the impulse responses, roughly 50% to 60% of a US interest rate increase is reflected in SOE interest rate increases. It is important to note that these results are not driven by any unusual behavior of the data during the Great Recession period. If we let the sample end in 2007m12, instead of 2013m12, the results are hardly different, as shown in appendix A. Also, leaving any one country out of the sample does not change any of the conclusions.

While the theoretical model below is constrained to the polar cases of a key currency area (i.e. the US) and a small open economy, reality might be more nuanced, as argued above. Figure 2 shows what happens to our results when the medium sized countries (or currency areas) are added to the sample. The blue lines with star symbols show the impulse responses for the whole data set comprising all countries for which data are available, whereas the responses from the data set constrained to the undoubtedly small countries are added in black (without markers) for comparison (these black comparison lines are the same as those shown in figure 1). The first row in figure 2 shows the responses of the small and medium countries' interest rate to a US monetary policy shock as well as the exchange rate response. The second row shows the responses to a contractionary monetary policy shock outside the US, and plots the response of the US interest rate and again of the bilateral exchange rate. Comparing the blue starred lines (all countries) with the black unmarked ones (small countries only) reveals that the results are qualitatively the same, and quantitative differences are mostly modest. The SOE interest rate response to a US tightening is roughly as strongly positive as shown before in figure 1. The US interest rate response to a foreign monetary tightening is slightly larger in the more comprehensive data set, indicating that US monetary authorities might respond somewhat more positively to foreign interest rate increases if these are generated by central banks of the larger set of countries. However, in view of the scale of the response, the overall reaction remains very small.

Upon close inspection, the exchange rate response to a monetary tightening in the non-US economies (lower right panel) shows two more months of initial appreciation before depreciation sets in, compared to the small economy only case. While the difference is not large, it is suggestive of the fact that adding medium sized economies to the sample moves the exchange rate response more towards a delayed overshooting pattern that is (and remains, see upper right panel) typical for the dollar's response to a US tightening. In appendix A, we further demonstrate that if we run the VAR only including data of the US and the group of medium sized countries, the associated exchange rate shows indeed a more pronounced hump in response to a non-US interest rate shock, and thus a stronger deviation from UIP, even though far weaker (and not statistically significant) than for a US policy rate shock. Our interpretation is that the assets of the medium sized economies share, to a limited extent, some of the properties of US currency and assets.

All in all, we see the evidence as compatible with a pronounced asymmetry that seems to depend on the role of different currencies for international trade. For truly small open economies, UIP more or less holds, while for the US the evidence is strongly against UIP. The medium sized countries find themselves in between. In the subsequent sections, we show that this evidence can be explained by a theory which predicts that the liquidity value of assets in international trade, that itself depends on the country's role in world trade, as well as the interest rate responses to foreign policy rate shocks drive the asymmetries of the exchange rate response to monetary shocks.

3 The model

In this section, we develop a macroeconomic framework with a modified UIP condition that allows explaining the exchange rate dynamics presented in the empirical section. At the heart of the analysis is a liquidity premium on US treasury securities, which is – inter alia – affected by the stance of monetary policy. While the focus of the paper is on exchange rate dynamics induced by a modified UIP condition, the determination of the liquidity premium and, in particular, the role of monetary policy requires applying a general equilibrium framework. Specifically, we develop a model with the US as a large open economy and a small open economy. To endogenously derive a liquidity value of US treasuries, we specify the central bank's supply of money in the large economy as an asset exchange, i.e. an exchange of US dollars against eligible assets.[10] Investors are aware of the fact that treasury bills are eligible, such that the latter are valued differently from non-eligible assets. The return on US treasuries therefore differs by a liquidity premium from returns on other assets, including interest rates on assets of foreign origin, which are associated with currencies that are used less than the US dollar in international trade.

3.1 The domestic economy

We specify the domestic economy as a stylized model of the US economy, consisting of infinitely many households, firms, retailers, and a public sector. The domestic economy will be assumed to be large such that the foreign economy will not affect the equilibrium allocation and prices that are specific for the large economy (see Section 3.3).

Households There are infinitely many households $i \in [0, 1]$, who are infinitely lived and have identical endowments and identical preferences. They enter period t with holdings of money, $M_{i,t-1} \geq 0$, short-term treasury securities, $B_{i,t-1} \geq 0$, and foreign currency denominated bonds $B^*_{i,t-1} \geq 0$. They participate in open market operations before they enter the goods market and the asset market. In open market operations, money is supplied outright or under repurchase agreements (repos) against eligible securities.[11] We assume that only domestic treasury bills are

[10] This mechanism is also applied in Hoermann and Schabert (2014) who analyze the macroeconomic effects of central bank balance sheet policies.

[11] Both types of money supply are considered to ensure that $I_{i,t}$ is non-negative in equilibrium. Further details on the timing of events and the flow of funds can be found in Hoermann and Schabert (2014).

eligible, such that household i faces the following constraint:

$$I_{i,t} \leq B_{i,t-1}/R_t^m, \tag{1}$$

Though individual agents do in reality not acquire money via open market operations, condition (1) describes how the supply of reserves in the money market is constrained from an aggregate point of view. Given that reserves are typically traded among individual banks in the US at the federal funds rate, which is targeted by the US Federal Reserve and closely follows the repo rate for treasury repurchase agreements (see for example Bech et al. 2012), we assume that the repo rate R_t^m, i.e. the relative price of money in terms of eligible assets, is directly controlled by the central bank. After the money market is closed, households enter the market for consumption goods, where household $i's$ expenditures are restricted by the cash constraint

$$P_t c_{i,t} \leq I_{i,t} + M_{i,t-1}, \tag{2}$$

where P_t denotes the price level of total consumption $c_{i,t}$, which consists of home and foreign goods, $c_{i,t} = c_{i,t}^h + c_{i,t}^f$. Thus, domestic currency serves as means of payment for domestic and foreign goods. Since the focus is on nominal exchange rate dynamics, we simplify by assuming perfect substitutability between home and foreign produced goods in consumption and the law of one price, which states that $P_t^h = S_t P_t^f$ (where P_t^h and P_t^f denote the prices of the domestic and foreign good, respectively, and S_t denotes the nominal exchange rate) such that purchasing power parity holds, implying $P_t = P_t^h$.

In the asset market, household i receives payoffs from maturing assets and can reinvest in treasuries, household debt, foreign bonds, and money. Before the asset market opens, it can repurchase treasuries. The budget constraint thus reads

$$\begin{aligned} &(B_{i,t}/R_t) + M_{i,t} + S_t(B_{i,t}^*/R_t^*) + (R_t^m - 1)\, I_{i,t} + P_t c_{i,t} + P_t \tau_t \\ \leq\ & B_{i,t-1} + M_{i,t-1} + S_t B_{i,t-1}^* + W_t n_{i,t} + P_t \varphi_t, \end{aligned} \tag{3}$$

where W_t is the nominal wage rate, $n_{i,t}$ working time, τ_t lump-sum taxes, φ_t profits from firms, and R_t^* the interest rate on foreign bonds. Note that the term $(R_t^m - 1)\, I_{i,t}$ summarizes the costs of acquiring money from the central bank, where outright purchases reduce bond holdings by R_t^m per unit of money and the costs of money temporarily held under repos is given by $R_t^m - 1$.[12] Household i maximizes the expected sum of a discounted stream of instantaneous utilities u :

$$E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_{i,t}, n_{i,t}\right), \tag{4}$$

[12] Outside open market operations, the central bank leaves aggregate money supply unchanged, $\int_0^1 M_{i,t} di = \int_0^1 (M_{i,t-1} + I_{i,t} - M_{i,t}^R) di$.

where E_0 is the expectation operator conditional on the time 0 information set, $\beta \in (0,1)$ is the subjective discount factor, and the period utility function is $u(c_{i,t}, n_{i,t}) = (1-\sigma)^{-1}c_{i,t}^{1-\sigma} - \chi(1+\omega)^{-1}n_{i,t}^{1+\omega}$ with $\sigma, \chi, \omega \geq 0$, subject to (1), (2), (3) and standard borrowing constraints, for given initial values $M_{i,-1}$, $B_{i,-1}$, and $B^*_{i,-1}$. The first order conditions for working time, consumption, additional money, as well as for holdings of government bonds, money, and foreign bonds are: $-u_{i,nt}/w_t = \lambda_{i,t}$,

$$u_{i,ct} = \lambda_{i,t} + \psi_{i,t}, \tag{5}$$

$$R^m_t\left(\lambda_{i,t} + \mu_{i,t}\right) = \lambda_{i,t} + \psi_{i,t}, \tag{6}$$

$$R_t\beta E_t\left[\left(\lambda_{i,t+1} + \mu_{i,t+1}\right)\pi^{-1}_{t+1}\right] = \lambda_{i,t}, \tag{7}$$

$$\beta E_t\left[\left(\lambda_{i,t+1} + \psi_{i,t+1}\right)\pi^{-1}_{t+1}\right] = \lambda_{i,t}, \tag{8}$$

$$R^*_t\beta E_t\left[(S_{t+1}/S_t)\lambda_{i,t+1}\pi^{-1}_{t+1}\right] = \lambda_{i,t}, \tag{9}$$

where π_t denotes the inflation rate $\pi_t = P_t/P_{t-1}$, w_t the real wage rate $w_t = W_t/P_t$, and $\mu_{i,t}$, $\psi_{i,t}$, as well as $\lambda_{i,t}$ the multiplier on the money market constraint (1), the goods market constraint (2), and the asset market constraint (3). Finally, the following complementary slackness conditions hold in the household's optimum *i.*) $0 \leq b_{i,t-1}\pi_t^{-1} - R^m_t i_{i,t}$, $\mu_{i,t} \geq 0$, $\mu_{i,t}(b_{i,t-1}\pi_t^{-1} - R^m_t i_{i,t}) = 0$, and *ii.*) $0 \leq i_{i,t} + m_{i,t-1}\pi_t^{-1} - c^h_{i,t}$, $\psi_{i,t} \geq 0$, $\psi_{i,t}(i_{i,t} + m_{i,t-1}\pi_t^{-1} - c^h_{i,t}) = 0$, where $m_{i,t} = M_{i,t}/P_t$, $b_{i,t} = B_{i,t}/P_t$, and $i_{i,t} = I_{i,t}/P_t$, as well as (3) with equality and associated transversality conditions. Relating the first order condition for domestic treasuries (7) to the first order condition for money holdings (8), and using (5) and (6) to substitute out the multipliers, shows that the treasury rate equals the expected policy rate up to first order,

$$1/R_t = \frac{E_t\left[\left(1/R^m_{t+1}\right)\cdot\left(u_{i,ct+1}/\pi_{t+1}\right)\right]}{E_t\left(u_{i,ct+1}/\pi_{t+1}\right)}. \tag{10}$$

A close association between US policy and treasury rates is in accordance with empirical evidence (see appendix A). A comparison of (7) with the first order condition for foreign bonds (9), which are not eligible in domestic open market operations, shows that the long-run (real) treasury rate R can be smaller than the long-run rate of return on foreign bonds R^* (for $lim_{t\to\infty}S_{t+1}/S_t = 1$), if domestic treasuries exhibit a liquidity value, which is measured by the multiplier μ_t on the money market constraint (1).[13] We therefore interpret this spread as a liquidity premium.

Firms The production sector is standard. There is a continuum of monopolistically competitive intermediate producers indexed with $j \in [0,1]$. Intermediate goods are purchased by perfectly

[13]Treasury bill rates are only available for relatively few countries other than the US in the IFS data base. These countries are Belgium, Canada, France, Germany, Italy (from 1977m3 onwards), Japan, Spain (from 1979 onwards), and the UK. For these countries, the sample mean of the treasury bill rate minus annualized monthly CPI inflation, is 2.16%. For the US, the corresponding figure is 1.23% (these results change only little, to 1.99% vs. 1.10% if we measure inflation relative to the same month in the previous year).

competitive bundlers, who bundle/produce the final domestic consumption good y_t according to $y_t^{\frac{\epsilon-1}{\epsilon}} = \int_0^1 y_{jt}^{\frac{\epsilon-1}{\epsilon}} dj$, leading to a demand $y_{jt} = (P^h_{jt}/P^h_t)^{-\epsilon} y_t$, with $(P^h_t)^{1-\epsilon} = \int_0^1 (P^h_{jt})^{1-\epsilon} di$ (P^h_{jt} and P^h_t being the price of good j and the aggregate price level for domestic goods). Intermediate goods producing firms produce the amount y_{jt} applying the technology $y_{jt} = a_t n_{jt}$, where labor productivity a_t follows an exogenous first order autoregressive process. Labor demand thus satisfies: $mc_{jt} = w_t(P_t/P^h_{jt})/a_t$, where mc denotes real marginal costs. Staggered price setting forces a measure $\phi \in [0,1)$ of firms to adjust the previous period price with average inflation, while the measure $1-\phi$ chooses new prices $\overline{P}^h_{jt}$ as the solution to $\max_{\overline{P}^h_{jt}} E_t \sum_{s=0}^{\infty} \phi^s q_{t,t+s}(\overline{P}^h_{jt} y_{jt+s} - P^h_{t+s} mc_{t+s} y_{jt+s})$, s.t. $y_{jt+s} = (\overline{P}^h_{j,t})^{-\epsilon}(P^h_{t+s})^{\epsilon} y_{t+s}$, where $q_{t,t+s}$ is the stochastic discount factor of the owners (i.e. of households). The first order condition for their price $\overline{P}^h_{jt}$ is given by $Z_t = \frac{\epsilon}{\epsilon-1} Z_{1,t}/Z_{2,t}$, where $Z_t = \overline{P}^h_{jt}/P^h_t$, $Z_{1,t} = c_t^{-\sigma} y_t mc_t + \phi\beta E_t(\pi^H_{t+1}/\pi^H)^{\epsilon} Z_{1,t+1}$, $Z_{1,t} = c_t^{-\sigma} y_t + \phi\beta E_t(\pi^H_{t+1}/\pi^H)^{\epsilon-1} Z_{2,t+1}$ and $\pi^H_t = P^h_t/P^h_{t-1}$. Using the demand constraint, we obtain $1 = (1-\phi)\, Z_t^{1-\epsilon} + \phi(\pi^H_t/\pi^H)^{\epsilon-1}$. Given that aggregate labor input is $n_t = \int_0^1 n_{jt} dj$ and $n_{jt} = (P^h_{jt}/P^h_t)^{-\epsilon} y_t$, aggregate domestic output depends on the price dispersion, $y_t = a_t n_t / s_t$, where $s_t \equiv \int_0^1 (P^h_{j,t}/P^h_t)^{-\epsilon} dj$ and $s_t = (1-\phi) Z_t^{-\epsilon} + \phi s_{t-1}(\pi^H_t/\pi^H)^{\epsilon}$ given s_{-1}.

Public sector The government issues short-term nominally risk-free bonds B^T_t, which are either held by domestic households B_t, foreign households B^f_t, or the central bank B^C_t. We assume that the supply of short-term treasuries is exogenous and we assume that it follows a constant growth rate

$$B^T_t = \gamma B^T_{t-1}, \tag{11}$$

where $\gamma > \beta$. To avoid further effects of fiscal policy, we assume that the government can raise tax revenues in a non-distortionary way, $P_t \tau_t$, such that the government budget constraint is given by $\left(B^T_t / R_t\right) + P_t \tau^m_t + P_t \tau_t = B^T_{t-1}$, where $P_t \tau^m_t$ denotes central bank transfers.

The central bank supplies money in exchange for domestic treasuries in form of outright sales/purchases M_t and repurchase agreements M^R_t. At the beginning of each period, the central bank's stock of treasuries equals B^C_{t-1} and the stock of outstanding money equals M_{t-1}, it then receives an amount $R^m_t I_t$ of treasuries in exchange for money I_t, and after repurchase agreements are settled its holdings of treasuries reduces by B^R_t and the amount of outstanding money by $M^R_t = B^R_t$, such that its budget constraint reads $\left(B^C_t/R_t\right) + P_t \tau^m_t = (I_t/R^m_t) + B^C_{t-1} - B^R_t + M_t - M_{t-1} - \left(I_t - M^R_t\right)$. In accordance with common central bank practice, we assume that the central bank transfers interest earnings to the government, $P_t \tau^m_t = B^C_t (1 - 1/R_t) + (R^m_t - 1)\left(M_t - M_{t-1} + M^R_t\right)$, and that it rolls over its maturing assets. Substituting out $P_t \tau^m_t$ and I_t with $I_t = M_t - M_{t-1} + M^R_t$, in the budget constraint, shows that central bank holdings of treasuries evolve according to $B^C_t - B^C_{t-1} = M_t - M_{t-1}$. Further restricting the initial values B^C_{-1} and M_{-1} to satisfy $B^C_{-1} = -M_{-1}$, we get the central bank balance sheet constraint

$$B^C_t = M_t. \tag{12}$$

Following large parts of the literature, we assume that the central bank sets the policy rate according to a simple feedback rule

$$R_t^m = \left(R_{t-1}^m\right)^{\rho} (R^m)^{1-\rho} (\pi_t/\pi)^{\rho_\pi(1-\rho)} (y_t/y)^{\rho_y(1-\rho)} \exp(\varepsilon_{r,t}), \tag{13}$$

where $R^m > 1$, $\rho \geq 0$, $\rho_\pi \geq 0$, and $\rho_y \geq 0$, and $\varepsilon_{r,t}$ is a normally distributed i.i.d. random variable with $E_{t-1}\varepsilon_{r,t} = 0$. The central bank further sets an inflation target, which is consistent with the long-run inflation rate and satisfies $\pi > \beta$. To give a preview, we set the growth rate of T-bills γ equal to the central bank's inflation target, $\gamma = \pi$, which for the US accords to the estimated growth rate of T-bills (corrected by GDP growth) for the sample period 1966-2007. Finally, the central bank sets the ratio of money supplies under both types of open market operations $\Omega : M_t^R = \Omega \cdot M_t$.

3.2 The foreign economy

Like the domestic economy, the foreign economy – which we model as a small open economy whose variables will have no influence on domestic variables – consists of infinitely many households, firms, retailers, all being of mass one, and a public sector. Production and price setting in the foreign economy is assumed to correspond to production and price setting in the domestic economy, and foreign households exhibit the same preferences as domestic households. Foreign households supply working time n_t^*, consume domestic and foreign goods, $c_t^* = c_t^{*,h} + c_t^{*,f}$, and have access to domestic and foreign assets. We assume that foreign households also assign a positive transaction value to domestic currency, as it serves as a key currency for international trade in goods and assets (see also Canzoneri et al., 2013b).

Accordingly, foreign agents hold domestic treasuries not only as they provide a store of value, but also for transaction purposes. For domestic residents of the key currency economy, we modelled the liquidity value of treasuries as deriving from them giving access to the central bank's open market operations (see 1). In principle, we could assume the same for foreign country residents. However, in reality the liquidity value of treasury bonds might also stem from other more indirect channels. We therefore model the liquidity value of large country treasuries in a reduced form way by assuming that holding them lowers transaction costs. Thus, abstracting from further details associated with modelling foreign money supply and transactions explicitly, we introduce a simple transaction cost function h to account for transaction services of domestic treasury securities (see Lahiri and Vegh, 2003, or Linnemann and Schabert, 2010). Notably, by applying a transaction costs function we further avoid a non-stationarity that would otherwise be induced by the current account. Specifically, denoting by $B_t^{*,f}$ and B_t^f the foreign country's holdings of foreign and domestic bonds, we assume that transaction costs $h_t = h(c_t^*, B_{t-1}^{*,f}/P_t^*, B_{t-1}^f/(S_t P_t^*))$ are non-negative, increasing in total foreign consumption c_t^*, strictly decreasing in the real value $B_{t-1}^f/(S_t P_t^*)$ of domestic treasury securities, $h_b < 0$, and – solely for consistency – also in the real value $B_{t-1}^{*,f}/P_t^*$ of foreign treasury securities, $h_{b^*} < 0$. The transaction cost function is further twice continuously differentiable in all arguments, and satisfies $h_{cc} \geq 0$, $h_{bb} > 0$, $h_{b^*b^*} > 0$, and is separable in all arguments.[14] Analogously to domestic households, a representative foreign households maximizes $E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t^*, n_t^*\right)$, subject to the budget constraint

$$\left(B_t^{*,f}/R_t^*\right) + \left(1/S_t\right)\left(B_t^f/R_t\right) + P_t^* c_t^* + P_t^* h_t \leq P_t^* w_t^* n_t^* + B_{t-1}^{*,f} + \left(1/S_t\right) B_{t-1}^f + P_t^* \Lambda_t^*, \quad (14)$$

where R_t^* denotes the foreign monetary policy rate, P_t^* the foreign consumption price (with $P_t^* = P_t^f = P_t^h/S_t$) and Λ_t^* collects real foreign profits and transfers. The first order conditions for consumption, working time, foreign and domestic treasuries can be summarized by $-u_{nt}^*\left(n_t^*\right)/v(c_t^*) = w_t^*$,

$$\beta E_t \left[v(c_{t+1}^*) \left(1 - h_b(b_t^f/\pi_{t+1})\right) / \pi_{t+1} \right] = v(c_t^*)/R_t, \quad (15)$$

and $\beta E_t[v(c_{t+1}^*)(1 - h_{b^*}(b_t^{*,f}/\pi_{t+1}^*))/\pi_{t+1}^*] = v(c_t^*)/R_t^*$, where $\pi_t^* = P_t^*/P_{t-1}^*$ and $v(c_t^*) = u_{ct}^*/(1 + h_c(c_t^*))$. The production technology of individual firms $k \in [0,1]$ satisfies $y_{k,t}^f = n_{k,t}^*$, such that profit maximizing labor demand is given by $P_t^f mc_t^* = P_t^* w_t^*$. As prices are set by retailers in a staggered way (like in the domestic economy), foreign inflation satisfies $1 = (1-\phi)\left(Z_{1,t}^*/Z_{2,t}^*\right)^{1-\varepsilon} + \phi\left(\pi^*\right)^{1-\epsilon}\left(\pi_t^*\right)^{\epsilon-1}$, where $Z_{1,t}^* = \left[\epsilon/\left(\epsilon-1\right)\right]\left(c_t^*\right)^{-\sigma} y_t^f w_t^* + \phi\beta\left(\pi^*\right)^{-\epsilon} E_t\left(\pi_{t+1}^*\right)^{\epsilon} Z_{1,t+1}^*$ and $Z_{2,t}^* = \left(c_t^*\right)^{-\sigma} y_t^f + \phi\beta\left(\pi^*\right)^{1-\epsilon} E_t\left(\pi_{t+1}^*\right)^{\epsilon-1} Z_{2,t+1}^*$. Aggregate foreign production then satisfies $y_t^f = n_t^*/s_t^*$, where the price dispersion measure s_t^* evolves according to $s_t^* = (1-\phi)\left(Z_{1,t}^*/Z_{2,t}^*\right)^{-\epsilon} + \phi\left(\pi^*\right)^{-\epsilon} s_{t-1}^*\left(\pi_t^*\right)^{\epsilon}$.

The foreign public sector consists of a central bank and a treasury, which are both specified in a way that avoids unnecessary complexities. Instead of modelling foreign central bank open market operations, we assume that the central bank simply sets the interest rate on foreign treasuries according to a standard interest rate feedback rule

$$R_t^* = \left(R_{t-1}^*\right)^{\rho}\left(R^*\right)^{1-\rho}\left(\pi_t^*/\pi^*\right)^{\rho_\pi(1-\rho)}\left(y_t^f/y^f\right)^{\rho_y(1-\rho)} \exp(\varepsilon_{r,t}^*), \quad (16)$$

which corresponds to the interest rate rule of the domestic central bank (13) and exhibits the same feedback parameters. The treasury is assumed to have access to lump-sum taxes and to supply securities in a way that keeps the real value of beginning-of-period debt constant, such that the associated foreign household investment decision satisfies an almost conventional Euler equation in equilibrium.[15]

3.3 Equilibrium

Market clearance implies that aggregate resources are constrained by $y_t + y_t^f = c_t + c_t^*$ and $y_t^f = c_t^* + (b_t^f/R_t) - b_{t-1}^f \pi_t^{-1}$, and that holdings of domestic treasuries satisfy $b_t^T = b_t + b_t^C + b_t^f$. Given that we think of the domestic economy as the US, we assume that it is large in the sense that transactions

[14] We neglect any role of foreign currency for simplicity.

[15] Specifically, we let $h_{b^*} \to 0$ and $h_c \to 0$, which implies that the demand for foreign treasuries simplifies to $\beta E_t[u_c(c_{t+1}^*)/\pi_{t+1}^*] = u_c(c_t^*)/R_t^*$.

with the small economy are negligible for the determination of domestic macroeconomic aggregates and prices (and we will calibrate the model in Section 5 accordingly). As a consequence, aggregate resources and holdings of treasuries are for the domestic economy constrained by $y_t = c_t^h$ and $b_t^T = b_t + b_t^C$, respectively. One can then solve for the equilibrium allocation and the associated price system of the large economy independently of the small open economy, while stationarity of the latter is induced by the marginal transaction cost h_b. The full set of equilibrium conditions can be found in appendix B. It should be noted that the model exhibits the classical property of an indetermined level of the exchange rate, due to purchasing power parity. Throughout the subsequent analysis, we will therefore focus on the behavior of the rate of depreciation $E_t S_{t+1}/S_t$, satisfying (9).

4 Liquidity premia and exchange rates

In this section, we show how the existence of a liquidity premium and its response to changes in the domestic monetary policy rate can, in principle, affect the exchange rate response consistent with the empirical evidence. Throughout the subsequent analysis, we restrict our attention to the case where the domestic goods market constraint, $P_t c_t \leq M_t + M_t^R$, is binding, such that monetary policy is non-neutral. Combining (5) and (8) leads to $u_{ct} = \beta E_t(u_{ct+1}/\pi_{t+1}) + \psi_t$ in equilibrium, which can be written as $\psi_t/u_{ct} = 1 - 1/R_t^{Euler}$, where R_t^{Euler} is defined as $1/R_t^{Euler} = \beta E_t \frac{u_{ct+1}}{u_{ct}\pi_{t+1}}$ and will be called "Euler equation rate", following Canzoneri et al. (2007). An Euler equation rate larger than one thus indicates a positive valuation for money and implies $\psi_t > 0$, such that households will not hold more money than for consumption expenditures. Now consider the money market constraint (1), which in equilibrium reads

$$B_{t-1}/R_t^m \geq M_t - M_{t-1} + M_t^R. \tag{17}$$

Using (5), (6), and (8), shows that its multiplier μ_t satisfies $\mu_t/u_{ct} = (1/R_t^m) - (1/R_t^{Euler})$ in equilibrium. Hence, when the policy rate is smaller than the Euler equation rate, $R_t^m < R_t^{Euler}$, the multiplier is positive $\mu_t > 0$ and the money market constraint (17) is binding. In this case, the goods market constraint (2) is binding as well, $\psi_t > 0$, given that $R_t^m \geq 1$. When households get money in exchange for treasuries at a price, $R_t^m - 1$, which is below their marginal valuation of money, $R_t^{Euler} - 1$, they use treasuries to acquire money until (17) is binding. As a consequence, there exists a premium between treasuries and non-eligible assets which increases with the liquidity value of treasuries μ_t. When the policy rate increases, the price of money in terms of treasuries also increases, such that the liquidity value of treasuries falls. For a given value of the Euler equation rate, the liquidity premium is therefore negatively affected by the policy rate.

Combining the first order condition for treasuries (7) with the first order condition for foreign bonds (9), leads to the following arbitrage freeness condition, which relates to the uncovered interest rate parity condition:

$$\frac{E_t((S_{t+1}/S_t)(\lambda_{t+1}/\pi_{t+1}))}{E_t(\lambda_{t+1}/\pi_{t+1})} = \frac{R_t}{R_t^*} \cdot \frac{E_t\left[\left(1+\mu_{t+1}/\lambda_{t+1}\right)(\lambda_{t+1}/\pi_{t+1})\right]}{E_t(\lambda_{t+1}/\pi_{t+1})}, \tag{18}$$

and can, more compactly, be written as $\Gamma_t = (R_t/R_t^*) \cdot \Lambda_t$. According to (18) the term on the LHS, $\Gamma_t = \frac{E_t((S_{t+1}/S_t)(\lambda_{t+1}/\pi_{t+1}))}{E_t(\lambda_{t+1}/\pi_{t+1})}$, which – up to first order – equals the expected rate of depreciation, does not only depend on the spread between the domestic and the foreign interest rate, but is also affected by the liquidity premium $\Lambda_t = \frac{E_t\left[\left(1+\mu_{t+1}/\lambda_{t+1}\right)(\lambda_{t+1}/\pi_{t+1})\right]}{E_t(\lambda_{t+1}/\pi_{t+1})}$. Given that the latter is negatively related to the domestic policy rate, it tends to counteract the direct effect of the policy rate via the interest spread R_t/R_t^* (see RHS of 18). However, this effect of the liquidity premium might not be sufficient to change the qualitative prediction of UIP in a way that is consistent with empirical evidence. For this, the response of the foreign policy rate further matters as it determines the magnitude of changes in the interest rate spread R_t/R_t^* in response to a domestic monetary policy shock. To show this in an analytical way, we simplify the model by applying some specific parameter values, i.e. $\sigma = \gamma = \pi = 1$, $\rho_\pi = \rho_y = 0$, and $\Omega \to \infty$.[16] To examine the role of the foreign policy rate response to changes in the domestic policy rate and to allows for the observed co-movement of interest rates in Section 2, we assume – only for this section – that the foreign policy rate follows an apparently counterfactual rule (instead of 16), by which it only reacts to the domestic policy rate and to innovations $\varepsilon_{r,t}^* : R_t^* = R^*(R_t/R)^\kappa \varepsilon_{r,t}^*$. It can then easily be shown that the endogenous response of the liquidity premium together with a sufficiently large co-movement of the interest rates, which is governed by the parameter κ, can revert the prediction of a standard UIP condition with regard to the exchange rate response to US policy rate shocks. It should be noted that this result, which is summarized in the following proposition, will be confirmed numerically in the subsequent section applying a calibrated version of the model with standard interest rate rules (13) and (16) in which the interest rate comovement that is crucial for the result is not assumed but arises endogenously in equilibrium.

Proposition 1 *Consider a version of the model under a binding collateral constraint with $\sigma = \gamma = \pi = 1$, $\Omega \to \infty$, $\rho_\pi = \rho_y = 0$, and $R_t^* = R^*(R_t/R)^\kappa \varepsilon_{r,t}^*$. The liquidity premium decreases with the domestic policy rate, $\partial \Lambda_t / \partial R_t^m < 0$ if $\rho > 0$. Further,*

1. *an increase in the domestic policy rate leads to an increase in Γ_t and, up to first order, to an expected future appreciation (depreciation) if $\kappa > 1 - \rho$ (if $\kappa < 1 - \rho$), and*

2. *an increase in the foreign interest rate leads to an increase in Γ_t and, up to first order, to an expected future appreciation (depreciation).*

Proof. See Appendix C. ■

The results summarized in the proposition show that the existence of the liquidity premium and its endogenous reaction to an increase in the domestic policy rate, can revert the response of expected

[16] These parameters imply price stability, an exogenous domestic policy rate, and no outright money supply.

exchange rate changes compared to the standard UIP prediction. In contrast, a change in the foreign interest rate, which does not alter the liquidity premium on domestic treasuries, leads to a exchange rate response consistent with standard UIP. The condition presented in the part 1 of the proposition further shows that the co-movement between the foreign and the domestic interest rate is decisive for the exchange rate response. Only if κ is positive, such that the change in the interest rate spread is less pronounced than the change in the domestic interest rate, the endogenous response of the liquidity premium can lead to a reversal of the exchange rate dynamics. Since the US policy rate is empirically highly persistent, $1-\rho$ is a rather small quantity such that a limited and thus empirically plausible degree of co-movement κ suffices to fulfill the condition. In the subsequent Section, we abstain from the simplifying assumption on the foreign policy rate and consider the standard interest rule (16), by which the foreign monetary policy rate reacts to endogenously determined macroeconomic aggregates (i.e. foreign inflation and foreign output).

5 Numerical analysis

The above proposition showed the model's implications for exchange rate dynamics under the simplifying assumption of exogenous policy rates as well as for some other special parameter values chosen in order to be able to derive analytical results. Here, we present numerical evidence for a calibrated version of the model where the standard feedback rule (16) is applied for the foreign policy rate.

5.1 Parameter values

For the purposes of this section, we choose model parameters as follows. To avoid effects exclusively stemming from an asymmetric parameterization, we apply the same parameter values for both countries, except for those affecting their size. We specify for the intertemporal substitution elasticity of consumption and for the Frisch elasticity of labor supply $\sigma = \omega = 1.5$, which we consider a reasonable trade-off between diverging estimates resulting from microeconomic and macroeconomic data.[17] To ensure symmetric consumption demand functions, we neglect the marginal transaction cost of foreign consumption, $h_c \to 0$. The degree of price stickiness is chosen to match typical macro estimates and is set at $\phi = 0.75$ (an intermediate value lying between the estimates of Smets and Wouters (2007) and Justiniano and Preston (2010) for the US, which are between 0.65 and 0.90), and the absolute price elasticity is $\epsilon = 10$. We further choose χ to calibrate domestic working time in the steady state to equal $n = 0.33$. For the small open economy, we choose the weight on the disutility of labor such that steady state working time and output equal one percent of the US values. The steady state marginal transaction costs are further chosen so

[17] Card (1994) suggests a range of 0.2 to 0.5 for the Frisch elasticity while Smets and Wouters (2007) estimate $\omega = 1.92$. With respect to the intertemporal substitutability of consumption, Barsky et al. (1997) estimate an elasticity of 0.18 using micro data, implying a value of around 5 for σ. Macroeconomic data generally implies lower estimates, e.g. Smets and Wouters (2007) estimate $\sigma = 1.39$.

that the share of small open economy holdings of US treasuries equals one percent.

Since the model is solved in a log-linearly approximated form, we need to specify the elasticity of marginal transaction costs h_b with respect to domestic bond holdings, which we denote as $\Psi = \frac{h_{bb}b}{1-h_b}$. For this marginal transaction cost elasticity, there is no direct empirical evidence. We proceed by assuming that the role of bonds in facilitating transactions should be relatively small, and set the elasticity to 0.05 as in Linnemann and Schabert (2010). It should however be noted that the exchange rate response is not strongly affected by Ψ; we show the quantitative dependence of the results in a sensitivity analysis below.

The parameters of the interest rate rules of both countries are taken from Mehra and Minton (2007), $\rho_\pi = 1.5$, $\rho_y = 0.78$, and $\rho = 0.73$, where for simulations we also follow their results in choosing the standard deviations of the innovations to the interest rate rules as 0.326 per cent. We assume that the logarithm of labor productivity of the large economy follows an AR(1) process with an autocorrelation coefficient ρ_a equal to 0.9. We set the standard deviation of the innovation to this process, $\varepsilon_{a,t}$, to a value such that the overall standard deviation of simulated output matches the standard value of 1.5 per cent for US data. This requires a standard deviation of $\varepsilon_{a,t}$ of 0.98 per cent.

The steady state inflation rate target π (equal to the growth of T-bills γ) and the long-run policy rate R^m are set to the 20-year averages of U.S. consumer price inflation and, respectively, the Federal Funds rate, $\pi = 1.00575$ and $R^m = 1.0105$. Given that the liquidity value of foreign treasury securities are irrelevant for the results, we set the mean of the foreign policy rate R^* equal to π^*/β, where $\pi^* = \pi$. Given that the beginning of period real value of foreign treasuries is held constant, we further apply $h_{b^*} \to 0$, for convenience. The parameter Ω is the share of domestic reserves supplied in repurchase operations to total reserves, which we set at $\Omega = 1$ (we checked that all results are robust to variations in this value). The discount factor β is calibrated to match a steady state liquidity premium of 65 basis points, which follows Canzoneri et al. (2007) who choose this value as the empirical average difference between the interest rate for high-quality (AAA) borrowers and the interest rate on 3 months treasury bills. Thus, the discount factor is set to $\beta = \frac{\pi}{R^m + 65 \cdot 10^{-4}} = 0.9889$.

5.2 Numerical results

We present impulse responses to a shock to the disturbance $\varepsilon_{r,t}$ in the monetary policy rule based on a log-linear approximation of the model. Our focus is on exchange rate dynamics, such that we show the responses of the variables entering the modified interest rate parity condition (18). Thus, Figure 3 shows the percentage responses of the domestic treasury rate R_t, the foreign policy rate R_t^*, the interest rate differential $R_t - R_t^*$, the liquidity premium Λ_t, and the expected nominal depreciation rate $E_t S_{t+1}/S_t$ to a US monetary shock that raises the US policy interest rate R_t^m by one percentage point.

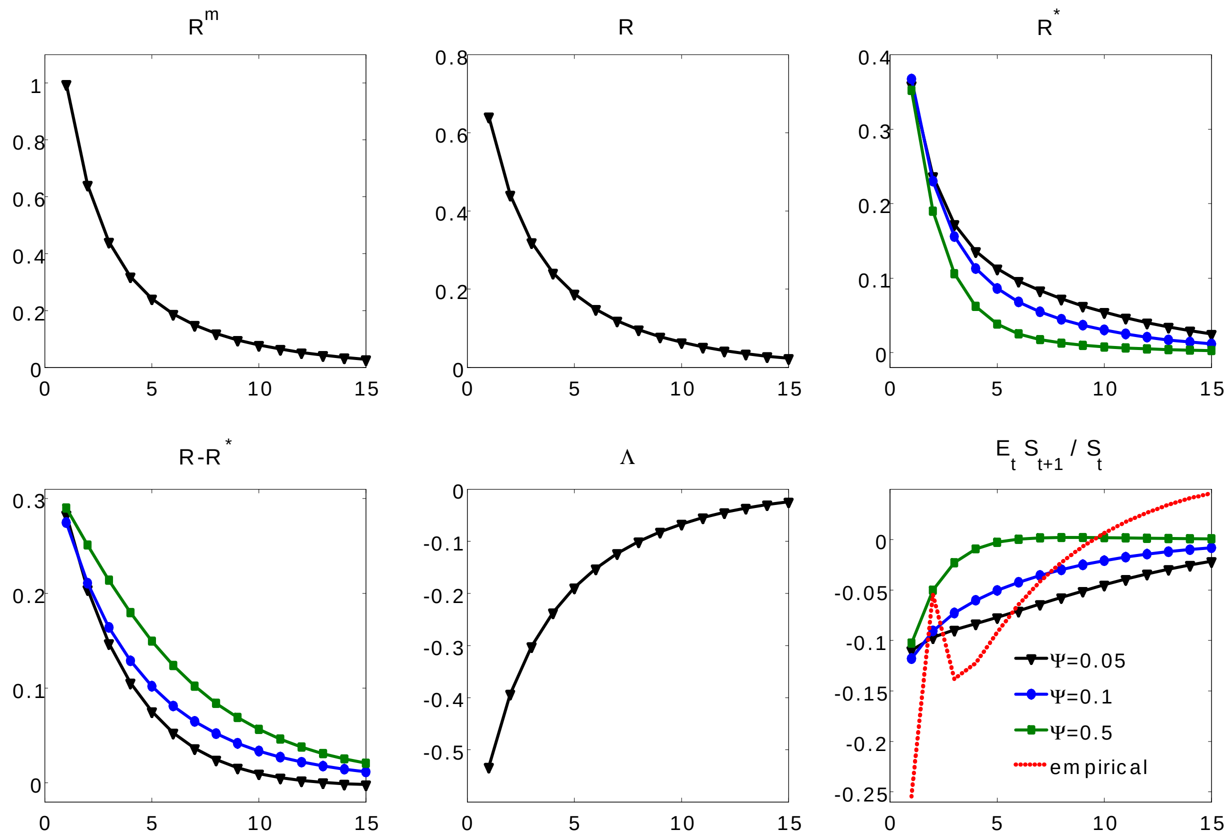

Figure 3: Impulse responses to one percentage point shock to domestic policy interest rate R^m (black/triangles: $\Psi = 0.05$, blue/circles: $\Psi = 0.1$, green/squares: $\Psi = 0.5$, red line without markers in lower right panel: empirical response)

As argued in Section 2, the empirical results shown in Figure 1 point out that typically the foreign interest rate does change positively in response to a shock to US policy rates. This is important in the present context since ignoring this international interest rate linkage would lead us to overstate the consequences of US interest rate shocks on interest rate differentials, which are decisive for exchange rate dynamics determined by (18), in particular, when, as in our model, endogenous changes in the liquidity premium tend to move the exchange rate in a different direction (see proposition 1). The theoretical impulse responses in Figure 3 for the benchmark parameterization are drawn in black with triangles. They show that after a US monetary policy shock that raises the policy interest rate R_t^m, the foreign policy rate R_t^* and the treasury bill rate R_t rise, too (see upper middle panel). These interest rate responses are well in line with empirical evidence (see section 2 and appendix A). Since the response of the treasury rate is more pronounced than the response of the foreign policy rate, there is an increase in the international interest rate differential $R_t - R_t^*$ (shown in the lower left panel). The liquidity premium Λ_t (lower middle panel) declines, as described in Section 4. From the modified interest rate parity condition (18), all else equal, the increase in the interest rate differential would lead to a future depreciation, while the decrease in the liquidity premium would lead to a future appreciation. However, the interest rate differential responds less than one for one to a contractionary US monetary shock because of the muted reaction of the treasury rate R_t (see 10) and the endogenous adjustment of the foreign interest rate R_t^*, which tends to increase with an increase in foreign inflation according to PPP together with a standard interest rate feedback rule (16) (see upper right panel). The interest rate differential response is then sufficiently small, such that its impact on the exchange rate can be dominated by the decrease in the liquidity premium leading to an expected exchange rate appreciation of the domestic currency (a negative response of $E_t S_{t+1}/S_t$). Note that the behavior of the interest rate differential in the full model discussed here endogenously fulfills the necessary condition stated in proposition 1 in the context of the simplified model version. In this way, the model is able to explain the response of the depreciation rate that is observed in the data.

In the right panel of the second row in Figure 3, the red dotted line is the corresponding empirical estimate of the depreciation rate following a US interest rate shock for comparison. Recall that in the empirical models we estimated above, as common in the empirical literature, the nominal (log) exchange rate entered as a variable, whereas in the theoretical discussion here we focus on the slope of the exchange rate response, namely the rate of depreciation. In Figure 3, the red dotted line represents the depreciation rate that is implicit in our empirically estimated log exchange rate response, obtained by converting the empirical response as shown in Figure 1 to its quarterly equivalent and then taking the forward difference. The overall pattern of the empirical (red dotted) response of the depreciation rate is in line with the model's prediction. Although we emphasize that the model as it stands is deliberately stylized and not suited to closely match the properties of data, its predictions are nonetheless in qualitative accordance with empirical observation. Figure 3 further shows that variations in the elasticity of the marginal transaction services, $\Psi \in \{0.05, 0.1, 0.5\}$, leave this conclusion qualitatively unchanged. Even an increase in the elasticity by the factor 10 leads to a similar, though less pronounced, response of the exchange rate.

The reason for the influence of Ψ is that the domestic monetary policy shock, by the logic of PPP and an active interest rate rule, raises both real interest rates and thus inflation in the foreign economy, such that the foreign interest rate increases. This is accompanied by a worsening of the current account such that foreign residents hold less domestic treasuries. This tends to increase transaction costs according to (15), which mitigates the initial expansionary effect on the foreign economy. Thus, the increase in R_t^* is less pronounced for higher values of the elasticity of the marginal transaction services.

Figure 4 shows the response to a negative autocorrelated shock to the foreign (SOE) interest rate R_t^* leading to a one percentage point decrease in the small economy's interest rate in the

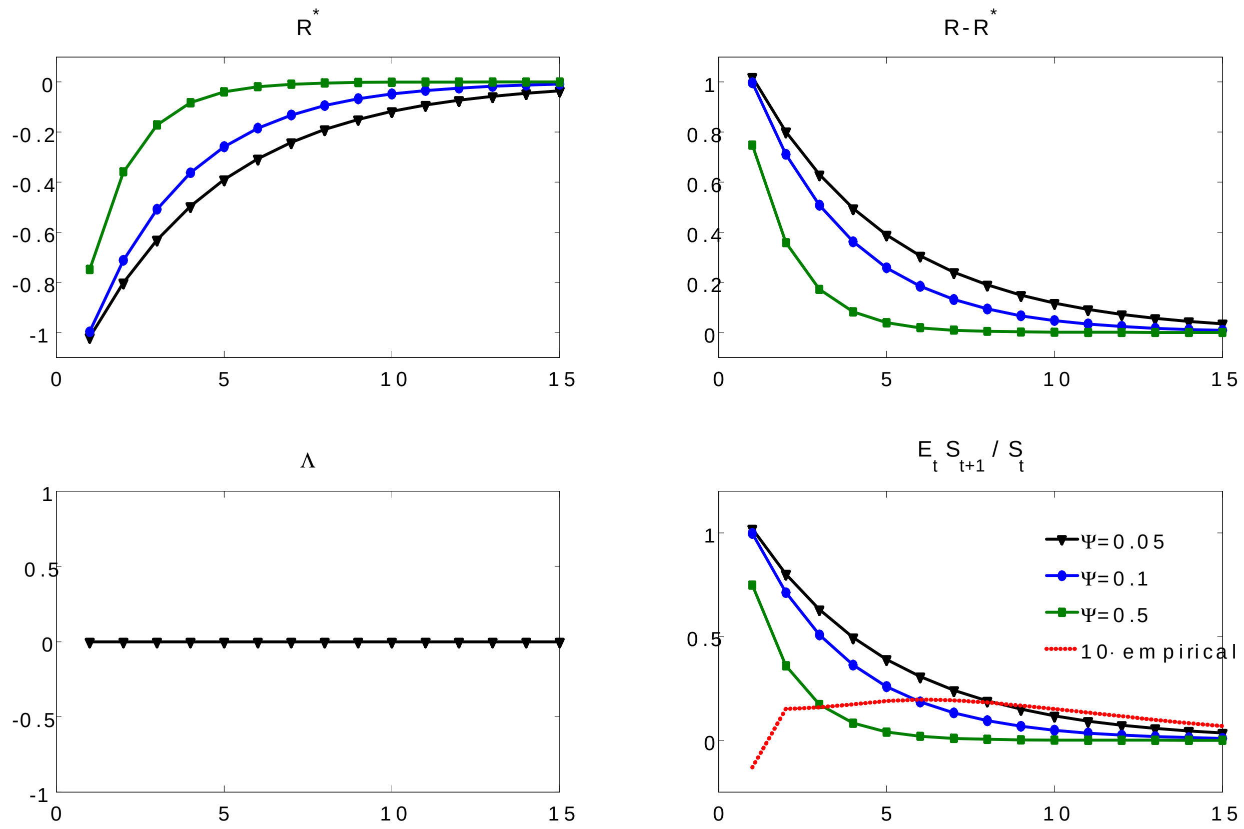

Figure 4: Responses to one percentage point decrease in foreign (SOE) policy interest rate (black/triangles: $\Psi = 0.05$, blue/circles: $\Psi = 0.1$, green/squares: $\Psi = 0.5$, red line without markers in lower right panel: empirical response)

baseline case ($\Psi = 0.05$, with a sensitivity analysis as above displaying results for $\Psi = 0.1$ and $\Psi = 0.5$). By construction, the domestic economy is large, such that the foreign interest rate shock does not affect domestic variables in general and the liquidity premium in particular. Hence, the exchange rate response is dominated by the resulting interest rate differential $R_t - R_t^*$ (shown in the upper right) that (since the domestic treasury rate is unaffected) reflects the increase in R_t^* with the opposite sign. Thus, in accordance with the standard UIP prediction, the increase in the differential $R_t - R_t^*$ leads to a subsequent depreciation of the domestic currency as shown in the lower right panel. Again, we add (as the red dotted line in the lower right panel) the expected depreciation response implied by the empirical estimates. The sign of the response of the depreciation rate is consistent with what we observed in the empirical analysis, whereas the magnitude of the response is clearly hugely overstated, a property that our model shares with all models that imply a standard UIP condition (for better visibility, the empirical depreciation response is scaled by the factor 10 in the figure).

As revealed by the blue line with circles and the green line with squares in Figure 4, a higher value for the elasticity of the marginal transaction services Ψ tends to reduce the effect of the original shock to the feedback rule (16) on the foreign interest rate and on the exchange rate. The reason is that the decrease in R_t^* is accompanied by an improvement of the current account such that foreign agents hold more large country treasuries. Hence, transaction costs tend to decrease, which by (15) increases aggregate demand and thus inflation in the foreign economy. The endogenous interest rate adjustment to the latter tends to counteract the impact of the original shock, such that the decline in R_t^* is mitigated. This effect is more pronounced, and thus the impact on the exchange rate is dampened, for higher values of the elasticity of the marginal transaction services. Overall, the responses shown in the Figures 3 and 4 indicate that the model can indeed replicate the main qualitative results regarding the exchange rate responses to monetary policy shocks as reported in the empirical analysis.

Notably, the empirical literature has shown that the UIP prediction fails not only conditional on monetary policy shocks, but also unconditionally. This is evidenced in the kind of empirical tests conducted by Fama (1984) and many others surveyed in Froot and Thaler (1992) and Engel (2013). While we acknowledge that the model is too stylized to give a full account of unconditional data moments, we nevertheless briefly summarize results from running regressions on simulated model data. We generate artificial time series of length 500 and apply averages of 1000 runs. The empirical literature on UIP typically uses a regression of the following type, $E_t\widehat{S}_{t+1}-\widehat{S}_t = \delta_0+\delta(\widehat{R}_t-\widehat{R}_t^*)+\eta_t$, where δ_0 and δ are parameters to be estimated, η_t is stochastic disturbance term, and carets denote log-linearized terms, while our log-linearized modified interest rate parity condition (18) reads $E_t\widehat{S}_{t+1}-\widehat{S}_t = \widehat{R}_t-\widehat{R}_t^*+\widehat{\Lambda_t}$. The standard test of UIP in the empirical literature is to run the regression on empirical data and test the hypothesis that $\delta_0 = 0$ and $\delta = 1$. Using this procedure with the simulated data, we get an average regression coefficient δ of -0.5443 with an average standard error of 0.0177.[18] Recall that if we ran the same simulations in a model without liquidity premia, the estimated coefficient would be centered on 1, the value predicted by the standard UIP relation. In the present model where liquidity premia are present, the estimated coefficients are statistically significantly smaller than 1 due to the influence of the omitted liquidity premium variable Λ_t. Note that the empirical literature often finds negative coefficients even larger in absolute value (Froot and Thaler, 1992, report a mean estimate for δ of -0.8 over various studies). The result found here is due to the fact that foreign interest rate shocks lead to a partial effect that is consistent with UIP, and would thus (when taken in isolation) produce a δ coefficient of 1. The

[18]For the regressions, we use the realized depreciation rate $\widehat{S}_t - \widehat{S}_{t-1}$ from the model simulations and regress it on the model interest rate differential $\widehat{R}_{t-1} - \widehat{R}_{t-1}^*$, since this is the data that an econometrician not observing expectations would have to work with in empirical work with real world data. However, we emphasize that the results would change only very little if we took the model's true expected depreciation rate $E_t\widehat{S}_{t+1} - \widehat{S}_t$ and regressed it on $\widehat{R}_t - \widehat{R}_t^*$, instead.

overall regression coefficient that we estimate then reflects the combined effects of the modified UIP relation in the case of domestic monetary policy and productivity shocks and standard UIP dynamics in case of foreign interest rate shocks.

6 Conclusion

This paper examines the role of liquidity premia of assets denominated in a key currency for exchange rate dynamics. We apply a macroeconomic approach to liquidity premia on short-term treasuries originating from monetary policy implementation. The liquidity premium leads to a modification of uncovered interest rate parity (UIP), which contributes to explaining observed deviations from the latter. Specifically, the endogenous reaction of the liquidity premium to interest rate changes can lead to a future appreciation in response to an increase in the key currency policy rate when foreign policy rates also increase to a sufficiently large extent.

We provide empirical evidence that this pattern is particularly relevant for changes in interest rates on US treasuries, which are known to provide transaction services, both nationally and internationally. In contrast, our panel VAR analysis shows that changes in the interest rate of a small open economy leads to exchange rate responses that are consistent with UIP predictions. The liquidity premia approach presented in this paper thus helps to understand exchange rate responses to monetary policy shocks. However, since these arguably account for a limited fraction of the total variance of exchange rates, our theory does not provide a solution for the forward premium puzzle.

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Appendix

A Additional empirical results

In this appendix, we present some robustness exercises and further details on our empirical analysis. We first examine if including data on the Great Recession affects the results. Figure 5 compares the exchange rate response to a monetary tightening for different samples.

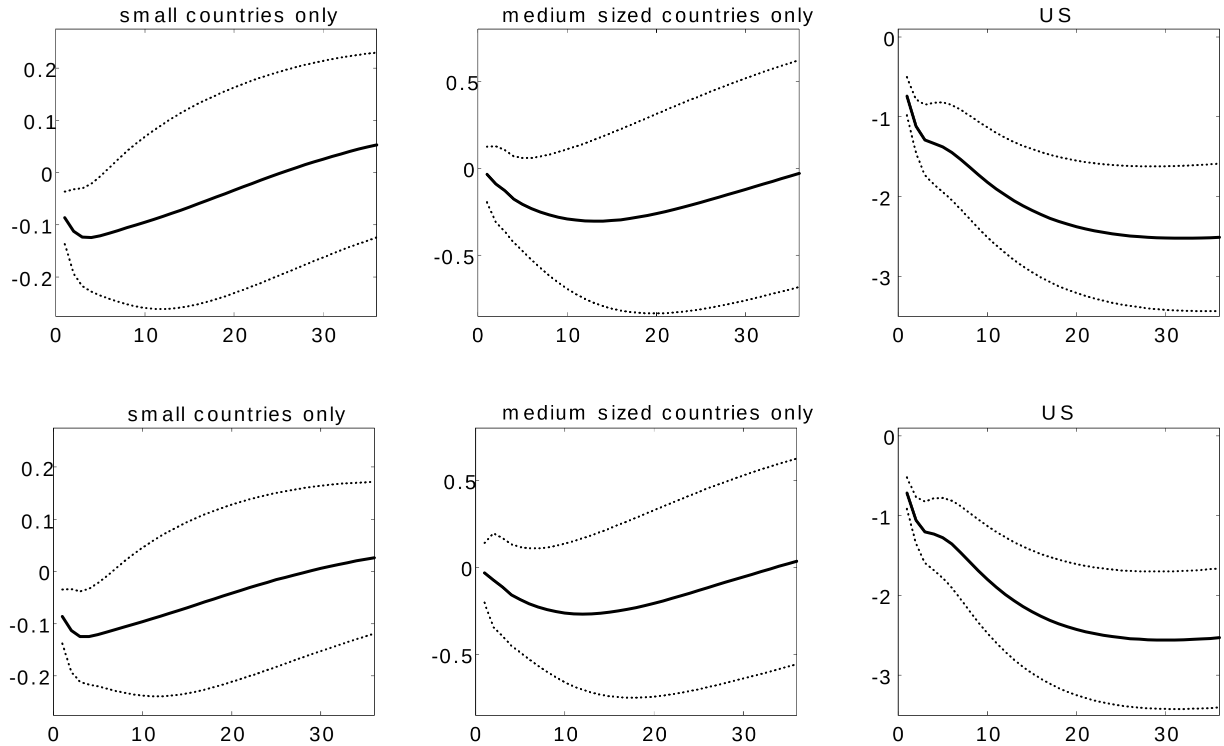

Figure 5: Impulse responses of exchange rate to monetary tightening for the full sample (upper row) and a pre-2008 sample (bottom row).

The upper row shows responses estimated with all available data in each case, i.e. up to 2013m12. The bottom row, for comparison, shows the results when the turmoil of the financial crisis and the ensuing Great Recession is left out by letting the samples end in 2007m12.

In each row, the respective left panel shows the result from the sample with the US and the group of small open economies only. It gives the response of the exchange rate vis-à-vis the US dollar to a positive interest rate shock in the small country group. The upper left panel shows the same response as the lower right panel of figure 1 in the main text. For comparison, the middle panel in each row shows the exchange rate response vis-à-vis the US dollar for a monetary tightening in the group of medium sized countries. This response is based on the estimated model containing data for the US and only the medium sized country group (Germany, UK, Euro area, France, Japan). The rightmost panel in each row shows, for the same data as the middle panel,

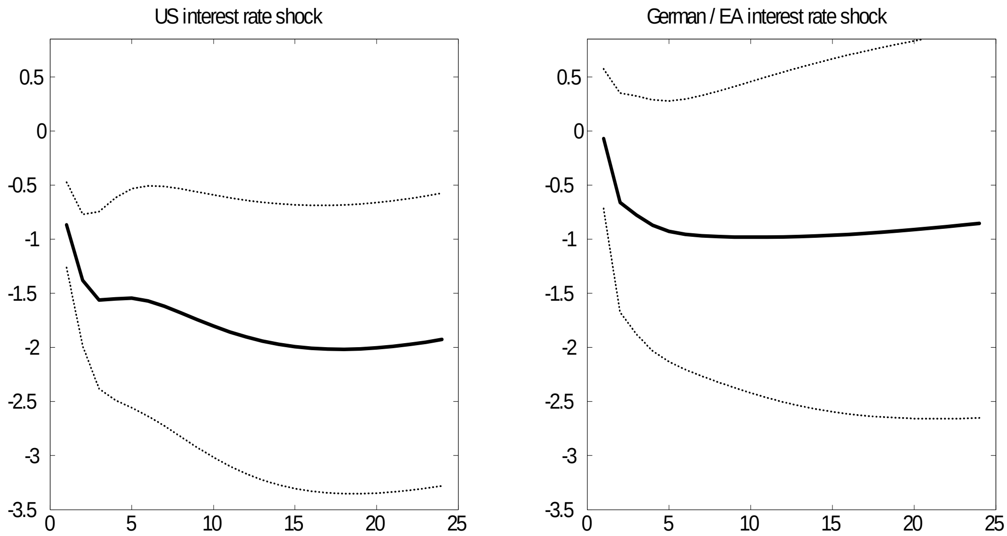

Figure 6: Exchange rate responses to monetary tightening in two-country VAR.

the corresponding exchange rate response to a US tightening (again, as in section 2, the exchange rate responses are shown such that a decline is a depreciation in the country, or country group, where the shock occurs).

The comparison underlines the observation mentioned in the main text: for truly small open economies (left panel), the response is roughly compatible with UIP, in that depreciation follows almost immediate upon initial appreciation. For the medium sized countries, there is evidence of a delay in overshooting, expressing itself in a pronounced hump that reaches a trough only after about a year. However, this appears statistically insignificantly different from zero at the 90 per cent level. The dollar's response to a US tightening, shown on the right, displays the familiar much stronger delayed overshooting response. These results are robust to constraining the sample period to the pre-Great Recession era (see second row of figure 5).

The behavior of the empirical estimates for the medium sized countries does not appear to be driven by particularities of the Euro area. For one thing, if we leave Euro area data out (and let the sample for Germany and France end in 1998m12, as in the panel VAR underlying Figure 2), the results are virtually unchanged. We also constructed a longer data set for Germany, by complementing the German data for inflation and industrial production for the whole sample (that is, 1975m1 up to 2013m12) by the Euro area's monetary policy interest rate (EONIA) and the Euro-Dollar exchange rate from 1999m1 onwards, and estimated a two-country VAR between the US and this prolonged German time series. The results, shown in figure 6, point out that there is some similarity in the exchange rate responses to a monetary tightening in the large country (US) and the medium sized country (Germany / EA). In both cases, there is evidence of delayed

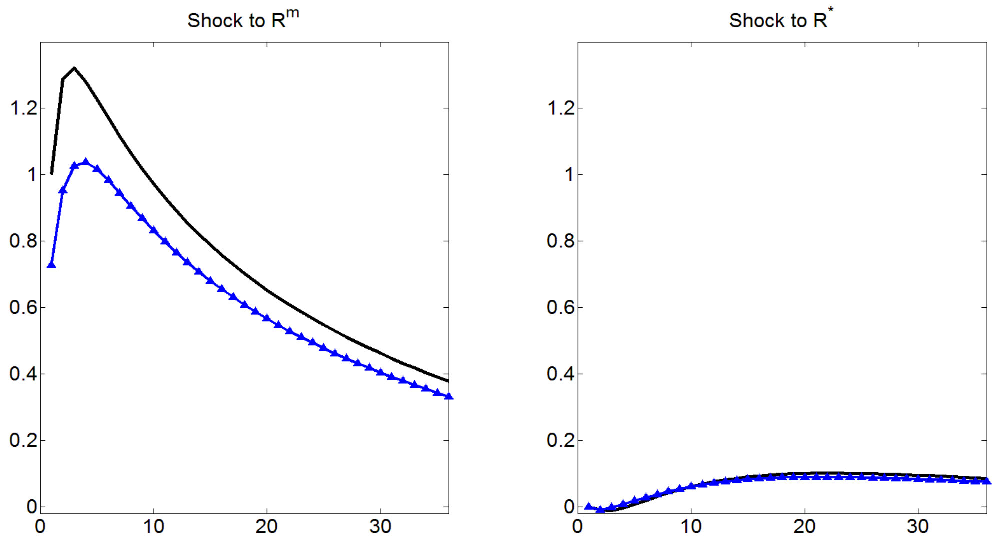

Figure 7: Empirical impulse responses to US (left panel) or other countries' (right panel) monetary policy shock. Solid lines without markers: R^m, triangles: R.

overshooting. However, for a shock originating in the US, it is more pronounced and statistically significantly different from zero.

Finally, we examine the relation between the US policy interest rate R^m (the Federal Funds Rate) and the US treasury bill rate R both under domestic and foreign monetary policy shocks (using the sample with all available country data, confining the sample to small economies only does not change the result in a noteworthy way). Figure 7 shows that the US treasury rate closely follows the Federal Funds rate, though it responds by less ($< 3/4$ in the first periods) than the Federal Funds rate after a domestic policy rate shock. In contrast, the responses of both rates to a foreign interest rate shock are almost indistinguishable.

B Equilibrium conditions for the large open economy

Given that the domestic economy is large, $c_t^{*,h} = 0$, $y_t = c_t^h$, $c_t = c_t^h$, and $b_t^T = b_t + b_t^C$ hold. We can then summarize the rational expectations equilibrium (REE) as the sets of sequences $\{c_t$, n_t, w_t, m_t, m_t^R, b_t, b_t^T, Z_t^1, Z_t^2, π_t, s_t, R_t, R_t^m, R_t^{Euler}, μ_t, $\lambda_t\}_{t=0}^{\infty}$ and $\{c_t^*$, n_t^*, w_t^*, b_t^f, $Z_{1,t}^*$, $Z_{2,t}^*$, π_t^*, s_t^*, R_t^*, $S_t/S_{t-1}\}_{t=0}^{\infty}$ satisfying

$$\chi n_t^{\omega}/w_t = \lambda_t, \tag{19}$$

$$\frac{\chi n_t^{\omega}}{w_t} = \beta E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}}, \tag{20}$$

$$E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1}} = R_t E_t \frac{c_{t+1}^{-\sigma}}{\pi_{t+1} R_{t+1}^m}, \tag{21}$$

$$1/R_t^{Euler} = \beta E_t \left[c_{t+1}^{-\sigma} / \left(c_t^{-\sigma} \pi_{t+1} \right) \right], \tag{22}$$

$$c_t = m_t + m_t^R, \text{ for } R_t^{Euler} > 1, \tag{23}$$

$$\text{or } c_t \leq m_t + m_t^R, \text{ for } R_t^{Euler} = 1$$

$$m_t + m_t^R = \frac{m_{t-1}}{\pi_t} + \frac{b_{t-1}/\pi_t}{R_t^m}, \text{ for } \mu_t > 0, \tag{24}$$

$$\text{or } m_t + m_t^R \leq \frac{m_{t-1}}{\pi_t} + \frac{b_{t-1}/\pi_t}{R_t^m}, \text{ for } \mu_t = 0,$$

$$m_t^R = \Omega m_t, \tag{25}$$

$$b_t = b_t^T - m_t, \tag{26}$$

$$b_t^T = \Gamma b_{t-1}^T / \pi_t, \tag{27}$$

$$\mu_t / \lambda_t = (R_t^{Euler} / R_t^m) - 1, \tag{28}$$

$$Z_t^1 = [\epsilon / (\epsilon - 1)] c_t^{-\sigma} y_t (w_t / a_t) + \phi \beta \pi^{-\epsilon} E_t \pi_{t+1}^{\epsilon} Z_{t+1}^1, \tag{29}$$

$$Z_t^2 = c_t^{-\sigma} y_t + \phi \beta \pi^{1-\epsilon} E_t \pi_{t+1}^{\epsilon-1} Z_{t+1}^2, \tag{30}$$

$$1 = (1 - \phi) \left(Z_t^1 / Z_t^2 \right)^{1-\varepsilon} + \phi \pi^{1-\epsilon} \pi_t^{\epsilon-1}, \tag{31}$$

$$y_t = c_t, \tag{32}$$

$$y_t = a_t n_t / s_t, \tag{33}$$

$$s_t = (1 - \phi) \left(Z_t^1 / Z_t^2 \right)^{-\epsilon} + \phi \pi^{-\epsilon} s_{t-1} \pi_t^{\epsilon}, \tag{34}$$

$$R_t^m = (R^m)^{1-\rho} \left(R_{t-1}^m \right)^{\rho} (\pi_t / \pi)^{\rho_\pi (1-\rho)} (y_t / y)^{\rho_y (1-\rho)} \exp(\varepsilon_{r,t}), \tag{35}$$

and

$$E_t \left[\frac{\lambda_{t+1} + \mu_{t+1}}{\lambda_t} \frac{R_t}{\pi_{t+1}} \right] = E_t \left[\frac{S_{t+1}}{S_t} \frac{\lambda_{t+1}}{\lambda_t} \frac{R_t^*}{\pi_{t+1}} \right], \tag{36}$$

$$S_t / S_{t-1} = \pi_t / \pi_t^*, \tag{37}$$

$$E_t \left[v(c_{t+1}^*) \left(1 - h_{b^*}(\widetilde{b}_t^f) \right) / \pi_{t+1} \right] = \left(R_t^* / R_t \right) E_t [v(c_{t+1}^*)(1 - h_b(b_t^f / \pi_{t+1}^*)) / \pi_{t+1}^*], \tag{38}$$

$$mc_t^* = -u_{nt}^* \left(n_t^* \right) / v(c_t^*), \tag{39}$$

$$Z_{1,t}^* = [\epsilon / (\epsilon - 1)] \left(c_t^* \right)^{-\sigma} y_t^f w_t^* + \phi \beta \left(\pi^* \right)^{-\epsilon} E_t \left(\pi_{t+1}^* \right)^{\epsilon} Z_{1,t+1}^*, \tag{40}$$

$$Z_{2,t}^* = \left(c_t^* \right)^{-\sigma} y_t^f + \phi \beta \left(\pi^* \right)^{1-\epsilon} E_t \left(\pi_{t+1}^* \right)^{\epsilon-1} Z_{2,t+1}^*, \tag{41}$$

$$1 = (1 - \phi) \left(Z_{1,t}^* / Z_{2,t}^* \right)^{1-\varepsilon} + \phi \left(\pi^* \right)^{1-\epsilon} \left(\pi_t^* \right)^{\epsilon-1}, \tag{42}$$

$$s_t^* = (1 - \phi) \left(Z_{1,t}^* / Z_{2,t}^* \right)^{-\epsilon} + \phi \left(\pi^* \right)^{-\epsilon} s_{t-1}^* \left(\pi_t^* \right)^{\epsilon}, \tag{43}$$

$$n_t^* / s_t^* = c_t^* + (b_t^f / R_t) - b_{t-1}^f \pi_t^{-1}, \tag{44}$$

$$R_t^* = \left(R_{t-1}^* \right)^{\rho} \left(R^* \right)^{1-\rho} \left(\pi_t^* / \pi^* \right)^{\rho_\pi (1-\rho)} (y_t^f / y^f)^{\rho_y (1-\rho)} \cdot \exp(\varepsilon_{r,t}^*), \tag{45}$$

and the transversality conditions, given $\widetilde{b}_t^f = \widetilde{b}^f > 0$, $\{a_t, \varepsilon_{r,t}, \varepsilon_{r,t}^*\}_{t=0}^{\infty}$, and initial values $M_{-1}^H > 0$, $B_{-1} > 0$, $B_{-1}^T > 0$, $s_{-1} \geq 1$, and $s_{-1}^* \geq 1$.

It should be noted that the sequences for the endogenous variables of the large open economy

$\{c_t,\ n_t,\ w_t,\ m_t,\ m_t^R,\ b_t,\ b_t^T,\ Z_t^1,\ Z_t^2,\ \pi_t,\ s_t,\ R_t,\ R_t^m,\ R_t^{Euler},\ \mu_t,\ \lambda_t\}_{t=0}^{\infty}$ can be determined by solving (19)-(35) and thus independently of (36)-(45). The sequences for the remaining endogenous variables $\{c_t^*,\ n_t^*,\ w_t^*,\ b_t^f,\ Z_{1,t}^*,\ Z_{2,t}^*,\ \pi_t^*,\ s_t^*,\ R_t^*,\ S_t/S_{t-1}\}_{t=0}^{\infty}$ can then subsequently be determined by solving (36)-(45).

C Appendix to section 4

Suppose that the average domestic policy rate and the inflation target satisfy $R^m < \pi/\beta$ and $\pi > \beta \Rightarrow R^{Euler} > 1$, where variables without time index denote steady state values. Then, the money market constraint (17) as well as the cash constraint (2) are binding in the steady state, given that their multiplier are strictly positive, $\mu = c^{-\sigma}[(1/R^m) - \beta/\pi] > 0$ and $\psi = c^{-\sigma}[1 - \beta/\pi] > 0$. Further assume that the foreign policy rate is given by $R_t^* = R^*(R_t/R)^{\kappa}\varepsilon_{r,t}^*$. For the parameter values $\sigma = \gamma = \pi = 1$, and $\Omega \to \infty$, a REE in a neighborhood of this steady state reduces a to set of sequences $\{c_t,\ n_t,\ w_t,\ m_t^R,\ b_t,\ mc_t,\ Z_t^1,\ Z_t^2,\ \pi_t,\ s_t,\ R_t^*,\ R_t,\ R_t^m,\ \mu_t,\ \lambda_t,\ E_t\frac{S_{t+1}}{S_t}\}_{t=0}^{\infty}$ and $P_0 > 0$ satisfying (28)-(35), $\chi n_t^{\omega} = c_t^{-1} w_t / R_t^{Euler}$,

$$\lambda_t = \beta E_t[c_{t+1}^{-1}/\pi_{t+1}], \tag{46}$$

$$1/R_t^{Euler} = \beta E_t\left[c_{t+1}^{-1}/\left(c_t^{-1}\pi_{t+1}\right)\right], \tag{47}$$

$$E_t\frac{c_{t+1}^{-1}}{\pi_{t+1}} = R_t E_t \frac{c_{t+1}^{-1}}{\pi_{t+1}R_{t+1}^m}, \tag{48}$$

$$c_t = m_t^R, \tag{49}$$

$$m_t^R = \frac{b_{t-1}/\pi_t}{R_t^m}, \tag{50}$$

$$b_t = b_{t-1}/\pi_t, \tag{51}$$

and the transversality conditions, $R_t^* = R^*(R_t/R)^{\kappa}\varepsilon_{r,t}^*$, a monetary policy setting $\{R_t^m \geq 1\}_{t=0}^{\infty}$ according to (13), given $\{a_t, \varepsilon_{r,t}, \varepsilon_{r,t}^*\}_{t=0}^{\infty}$ and initial values $B_{-1} > 0$, $B_{-1}^T > 0$, and $s_{-1} \geq 1$.

Proof of proposition 1. Consider the modified UIP condition (18), which can be written as

$$\Gamma_t = (R_t/R_t^*) \cdot \Lambda_t, \tag{52}$$

where $\Lambda_t = \frac{E_t\left[\left(1+\mu_{t+1}/\lambda_{t+1}\right)(\lambda_{t+1}/\pi_{t+1})\right]}{E_t(\lambda_{t+1}/\pi_{t+1})}$ and $\Gamma_t = \frac{E_t((S_{t+1}/S_t)(\lambda_{t+1}/\pi_{t+1}))}{E_t(\lambda_{t+1}/\pi_{t+1})}$. Combining (49) and (50) to $c_t = b_{t-1}/(R_t^m\pi_t)$ as well as (46) and (47) to $c_t\lambda_t = 1/R_t^{Euler}$, gives $\lambda_t = \pi_t(R_t^m/R_t^{Euler})/b_{t-1}$. Using the latter and (28) to substitute out the multipliers λ_{t+1} and μ_{t+1}, the terms Γ_t and Λ_t in (52) can be rewritten as

$$\Lambda_t = \frac{1}{E_t(R_{t+1}^m/R_{t+1}^{Euler})} \quad \text{and} \quad \Gamma_t = \frac{E_t((S_{t+1}/S_t)\,(R_{t+1}^m/R_{t+1}^{Euler}))}{E_t(R_{t+1}^m/R_{t+1}^{Euler})}. \tag{53}$$

Replacing consumption in (47) with $c_t = b_{t-1}/(R_t^m\pi_t)$, and substituting out b_t with (51), leads to $R_t^m/R_t^{Euler} = \beta E_t R_{t+1}^m$. Given that monetary policy satisfies $\rho_\pi = \rho_y = 0$, such that (35) simplifies to $R_t^m = (R^m)^{1-\rho}\left(R_{t-1}^m\right)^\rho \exp(\varepsilon_{r,t})$, the ratio R_t^m/R_t^{Euler} satisfies

$$R_t^m/R_t^{Euler} = (R_t^m)^\rho \cdot \beta(R^m)^{(1-\rho)} \exp[(1/2)var(\varepsilon_{r,t})]. \tag{54}$$

where we used that $E_t \exp(\varepsilon_{t+1})^x = \exp[(1/2)\left(x^2\right) var(\varepsilon_{r,t})]$. Hence, the terms in (53) can be simplified to

$$\Lambda_t = (1/R_t^m)^{\rho^2} \cdot \Phi_1, \tag{55}$$

$$\Gamma_t = E_t\left(S_{t+1}/S_t\right) + cov_t\left(S_{t+1}/S_t, \exp(\varepsilon_{r,t+1})^\rho\right) \exp[-\rho^2(1/2)var(\varepsilon_{r,t})], \tag{56}$$

where $\Phi_1 = (R^m)^{-(1+\rho)(1-\rho)}(1/\beta)\exp[-\left(1+\rho^2\right)(1/2)var(\varepsilon_{r,t})]$ is constant. Thus, (55) implies that Λ_t is strictly decreasing in the policy rate if $\rho > 0$, which establishes the first claim in the proposition.

Using $R_t^* = R^*(R_t/R)^\kappa \varepsilon_{r,t}^*$,, to substitute out the foreign policy rate R_t^* in (52), leads to

$$\Gamma_t = R_t^{1-\kappa}\left[(R^*/R^\kappa)\,\varepsilon_t^*\right]^{-1} \Lambda_t. \tag{57}$$

Now use the arbitrage freeness condition (48) and substitute out c_{t+1}^{-1} with $c_{t+1}^{-1} = \lambda_{t+1} R_{t+1}^{Euler}$ and λ_{t+1}/π_{t+1} with $\lambda_{t+1}/\pi_{t+1} = (R_{t+1}^m/R_{t+1}^{Euler})/b_t$, leading to $R_t = E_t R_{t+1}^m$ and – by using the policy rule (35) – to $R_t = E_t\left[(R_t^m)^\rho (R^m)^{1-\rho}\exp(\varepsilon_{r,t+1})\right]$, which can be used to substitute out R_t on the RHS of (57)

$$\Gamma_t = \left[(R_t^m)^\rho E_t\left[\exp(\varepsilon_{r,t+1})\right](R^m)^{1-\rho}\right]^{1-\kappa}\left[(R^*/R^\kappa)\,\varepsilon_t^*\right]^{-1} \cdot \Lambda_t.$$

Applying the expressions of Λ_t and Γ_t in (55) and (56), then gives

$$E_t\left(S_{t+1}/S_t\right) + \Theta_t = (R_t^m)^{-\rho(\kappa-(1-\rho))}\left(1/\varepsilon_t^*\right) \cdot \Phi_2, \tag{58}$$

where $\Phi_2 = (R^*/R^\kappa)(1/\beta)(R^m)^{(\rho-1)(\kappa+\rho)}\exp[-\left(\rho^2+\kappa\right)(1/2)var(\varepsilon_{r,t})]$ is constant and the term Θ_t depends on a potentially time varying second order term, $\Theta_t = cov_t\left(S_{t+1}/S_t, \exp(\varepsilon_{r,t+1})^\rho\right) \cdot \exp[-\rho^2(1/2)var(\varepsilon_{r,t})]$. Hence, (58) implies that if $\kappa > 1-\rho$ ($\kappa < 1-\rho$) an increase in the domestic policy rate leads to an appreciation (a depreciation) up to first order. An increase in the foreign interest rate that is induced by a increase in ε_t^*, is as well, up to first order, associated with an subsequent appreciation of the domestic currency. ■

D Additional numerical results

Figure 8 plots impulse responses to a one percent shock to the domestic policy interest rate from the loglinear approximation to the theoretical model for different degrees of responsiveness of the foreign monetary policy rule (16) where we rename ρ_π as ρ_π^* (all other parameters, including those of the domestic policy rule, are left unchanged).

The black lines marked with triangles correspond to the baseline parameterization where the interest rate rule in the foreign economy is the same as the domestic one. The blue lines marked

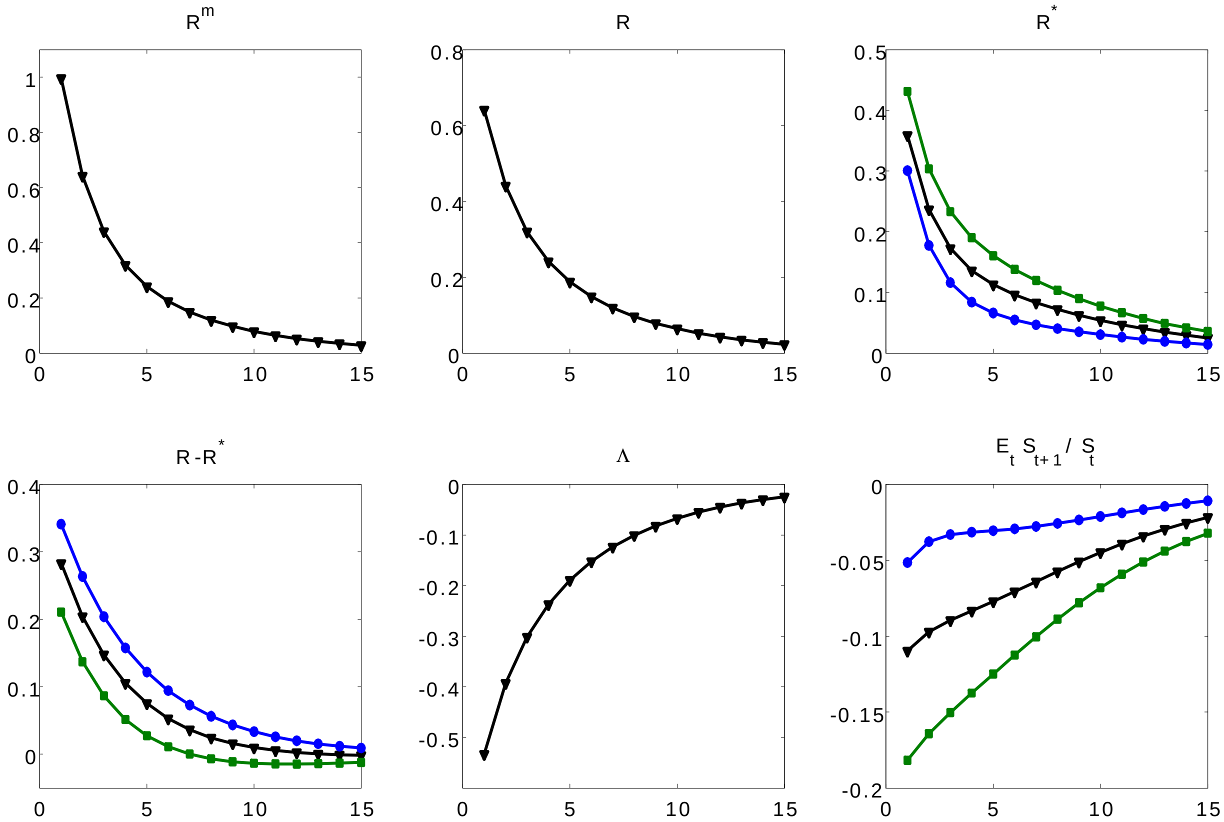

Figure 8: Model impulse responses to one percent shock to domestic policy interest rate: benchmark (black/triangles), more active SOE interest rate policy (blue/circles), less active SOE interest rate policy (green/squares)

with circles show the case of a more active interest rate policy in the small economy ($\rho_\pi^* = 2.5$), and the green lines marked with squares show the case of a less active interest rate policy in the small economy ($\rho_\pi^* = 1.1$). The figure confirms that both the interaction of the liquidity premium (lower middle panel) and the interest rate spread $R_t - R_t^*$ (lower left panel) jointly govern the response of expected depreciation (lower right panel). With a more (less) active foreign interest rate policy, the spread rises more (less) strongly. Consequently, the response of the expected depreciation rate is less (more) pronounced, though still negative, with a more (less) active foreign interest rate rule.